Mathematical Modelling of Flood and Debris Flows Caused by Outbursts from High Mountain Lakes

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Abstract

This paper discusses three models related to catastrophic hydrological occurrences in the mountains. The first model calculates and predicts the hydrographs of ice-dammed lake outbursts; the second model calculates the occurrence probability distributions of the volumes of water in and maximum discharges of water from moraine lakes; and the third one computes volume and discharge of flood water from the hydrograph, volume, and maximum discharge of the debris flow. The three models are developed for the high-elevation mountains where occurrences of glacial outbursts are quite common.

Introduction

Glacial outburst floods are quite rare at any particular site, but they are still quite common occurrences in high-elevation mountains. This paper presents principles for modelling of two dangerous hydrologic occurrences: outbursts from the ice-dammed lakes and from moraine lakes.

Outbursts from Ice-Dammed Lakes

Satellite photographs and glacial monitoring can be expected to provide sufficient information about the appearance and conditions of ice-dammed lakes. However, for forecasting purposes it is beneficial to have in addition a mathematical model of the hydrograph of the flood that can be expected from an eventual outburst.

We consider a process of discharge from the ice-dammed lake, starting from the beginning of the leakage in the ice-dam. The leakage process is defined by two main sources: the increasing cross-sectional area of the tunnel and the decline in the hydrostatic pressure due to the decreasing water volume in the lake. The assumption is that the potential energy of the water in the lake will be completely spent on the tunnel melt.

The equation for the relationship between the rate of expansion of the tunnel and decline of the water volume in the lake is as follows:

$$-\frac{d\omega}{dw} = \frac{\rho_0 [c(\theta_1 - \theta_2) + g(H + h) - \frac{v^2}{2}]}{\rho\, rl} \qquad (1)$$

where:

ω	is the cross-sectional area of the tunnel;
w	is the water volume in the lake;
$\rho = 1{,}000 kg/m^3$	is the density of water;
$\rho^{\cdot} = 920 kg/m^3$	is the density of ice;
r = 334,000 J/kg	is the latent heat of ice melting;
c = 4,190 J/(kg°C)	is the specific heat capacity of the water;
$g = 9.81\ m/s^2$	is the acceleration due to gravity;
$\theta_1 - \theta_2$	is the difference in temperature between the water in the lake and the water at the exit from the tunnel;
H	is the water level in the lake above the centre of weight the inflow hole of the tunnel;
h	is the difference in the elevations of inflow and outflow of the tunnel;
l	is the tunnel length; and
v	is the water velocity in the tunnel.

It is worth noting that $v^2/2$ in equation (1) is very small in comparison to the other values, and that the relationship between water level and water volume is expressed as follows:

$$H = a w^n \tag{2}$$

where,

a and *n* are the morphometric parameters of the lake depression.

We consider the process of outflow of water from a lake as the flow through a 'short pipe'.

$$Q = \alpha \omega^{\frac{5}{4}} \sqrt{H} \tag{3}$$

where,

Q	is the water flow discharge; and
α	is a coefficient evaluated from the best correspondence between the model and field observations.

First, we consider the case under the assumption that the lake water temperature is regulated by icebergs and the ice-dam and is equal to zero. Then the equation is:

$$Q = \alpha \{ \frac{\rho_0 g}{\dot{\rho} r l} [h(w_0 - w) + \frac{a}{n+1}(w_0^{n+1} - w^{n+1})] \}^{\frac{5}{4}} \sqrt{a w^n} \tag{4}$$

where,

w_0 is the initial water volume in the lake;

w is the variable water volume in the lake.

Using this expression, the maximum flood discharge is easily evaluated for a w computed from the following equation:

$$w_0\left(h + \frac{a}{n+1}w^n\right) = w\left[\left(\frac{2.5}{n} + 1\right)h + \frac{a}{n+1}\left(\frac{2.5}{n} + 3.5\right)w^n\right] \qquad (5)$$

Recomputation of the hydrograph **Q = f_1(w)** into Q = f_2(t) is done by differentiation. If the available information about the lake water temperature indicates $\theta > 0°C$ then we can use the following equation:

$$Q = \alpha\left\{\frac{\rho_0 g}{\rho' r l}\left[(h + x)(w_0 - w) + \frac{a}{n+1}(w_0^{n+1} - w^{n+1})\right]\right\}^{\frac{5}{4}}\sqrt{aw^n} \qquad (6)$$

$$x = \frac{C}{g}\theta\left[1 - \exp\left(-\frac{4000\alpha^{0.3} l(aw^n)^{0.15}}{Q^{0.55}\rho_0 C}\right)\right] \qquad (7)$$

It is obvious that equations (6) and (7) cannot be solved for Q. Therefore, equation (4) can be solved for the first approximation and then the principle of iteration can be applied. The coefficient a is defined by the tunnel length: for l = 0, α = 2.7; for l = 2km, α = 2.0; for l = 5km, α = 1.0; for l = 10km, α = 0.4; for l = 35km, α = 0.1; and for l = 50km, α = 0.07.

The model for computing outburst hydrographs with their characteristically strong negative asymmetry was validated for the following lakes: George in the Chugach Mountains, Alaska (1951), Talsequa on the Coastal Ridge on the border of British Columbia and Alaska (1958), the glacier Medvezhii in the western Pamir Mountains (1973), and Grenalown (1935, 1939) and Grimsvetn (1922, 1934, 1945), both in Iceland. Sufficient correspondence to the model was obtained with somewhat vague data for Lake Vatnsdalur (1898) in Iceland and the Pleistocene Missoula Lake in the Columbia River basin.

In cases when the tunnel diameter approaches dimensions corresponding to the size of the ice-dam, the model should be applied with great care. For example, at Lake Yapshan at the headwaters of the Shyok River in the Karakoram Mountains (1929), the cross-sectional area of the tunnel at the time of the maximum outburst flood is predicted to be equal to 65,500m^2 (diameter 290m), when the total height of the ice-dam was not greater than 150m (w_0 = 3.8*10^{9}m^3, h = 10m, l = 1,500m, n = 0.33, a = 0.08). It is obvious that when the tunnel diameter reached 50-70m, the tunnel vaults would collapse and the flow would be through an open channel.

Moraine Lake Outbursts

Moraine lakes are very common in the conditions of high-elevation mountains. They appear, evolve, and disappear. Sometimes they burst out catastrophically. Forecasting such outbursts is best accomplished by using graphs showing temporal increases in the water volumes in the lake. If such a curve has a parabolic shape, and the annual increment in the water volume increases consistently, the probability of an outburst becomes very high.

For purposes of ecological planning in mountainous terrain with moraine lakes, the probability distribution curves of lake volumes and maximum discharge rates are required. Methods for calculating such curves are not available. Nevertheless, there are some means to obtain such distribution curves by mathematical modelling under some reasonable assumptions. The following is one technique for such modelling — mainly stochastic, but with some elements of a deterministic approach.

We assume that the size of moraine lakes is dependent on the size of the glaciers nourishing them.

The parameters are:

- T is the age of the moraine lake (usually 50-100 years);
- C_v is a variance coefficient characterising the degree of possible deviation of the water volume in the lake resulting from random site-specific reasons (range 0.2-1.0);
- $\dot{p}$ is the probability of a moraine lake outburst during its maximum volume (range 0.2-0.5).

The average volume of a moraine lake during its maximum volume is computed as:

$$w_o = w_{oo} F^m \tag{8}$$

where:
$w_{00} = 0.2*10^6 m^3/km^{2m}$, $m = 0.7$, F is the glacier area in km^2.

The location of the point, t, along the time axis is defined as $t = \varepsilon_1 T$, where ε_1 is a uniformly distributed random value in the range of 0.0000 - 0.9999.

For the computation of w, a normally distributed random value, u_p, is first derived. The volume of the possible outburst is defined as:

$$w = \max\{w_o (\frac{t}{T})^s [1 + U_p C_v]; 0\}, \quad S = 5.6 \tag{9}$$

The probability of a lake outburst is computed as:

$$p = (p)^{\frac{w0}{w}} \tag{10}$$

The fact of outburst is defined as a uniformly distributed random value, ε_2, in the interval 0.0000 to 0.9999. The outburst occurs if $\varepsilon_2 < p$.

The maximum discharge of an outburst is computed as:

$$Q_m = K\,tg\alpha\,w^z, \quad K = 0.1,\ z = 0.75, \tag{11}$$

where,

α is the maximum average slope on a 100m section chosen on the moraine below the dam.

Iterative repetition ($N = 10^3 - 10^5$) of the described procedure forms synthetic rows, w and Q_m, which compose variation rows in decreasing order. Variation rows, w and Q_m, including zero values correspond to the order of probability computed from the equation $P = M/N$, where N is the number of cases accepted for the experiment and M is the order number of the variation row member.

The determined probabilities (P), together with the corresponding maximum discharges (Q_m) and the volumes (w), are used for building the distribution curves for these variables.

Parameters and numerical values of the coefficients are approximated on the basis of observations of moraine lakes in Pamir and Tian-Shan. The methodology of the modelling can be applied to any high-elevation mountain terrain, e.g., the Himalayas, Hindu Kush-Karakoram, Tibet, and the Andes.

Debris Flows

In many cases the proposed algorithms can be combined with the model of transport-dislocation debris flows. By this means, the characteristics of a given probability for debris flows can be obtained. This process can occur only on slopes that exceed the critical value:

$$tg\beta = \frac{(\rho - \rho_o)(1-\varepsilon)\,tg\,\varphi^{\cdot}}{[\rho(1-\varepsilon) + \rho_o\varepsilon]}, \tag{12}$$

where,

ρ is the rock density;

$\rho_o = 1{,}000 kg/m^3$ is the density of water;

ε is the porosity of the debris; and

$\varphi^{\cdot}$ is the dynamic angle of interior friction in the debris.

We assume that the increment of discharge of solid material (G) transported along with the debris flow is directly proportional to the coefficient of fluidity of the debris flow mass:

$$R = \frac{(\gamma_M - \gamma)}{(\gamma_M - \rho_o)}, \tag{13}$$

and the elementary potential power of the flow:

$$n = g \gamma Q_{DF} \sin\alpha, \tag{14}$$

and inversely proportional to the coefficient of debris mass stability on the slope (i.e.: $tg\,\varphi / tg\,\alpha$):

$$\frac{dG}{dl} = j \frac{tg\alpha}{tg\varphi} n R, \tag{15}$$

where,

- γ is the density of the debris mass;
- γ_M is the density of the debris mass at the limit of fluidity;
- φ is the static angle of interior friction;
- Q_{DF} is the debris flow discharge; and
- l is the length of flow movement along a channel with the slope $\alpha > b$.

Different transformations lead to the following equations:

$$G = N Q, \tag{16}$$

$$Q_{DF} = [1 + (1+\zeta) N] Q, \tag{17}$$

$$[1 + (\zeta + \frac{\rho}{\rho_o}) N]^{(\rho - \rho_o)/(\zeta\rho_o + \rho)} [1 + (\zeta - \zeta_M) N]^{(1+\zeta_M)/(\zeta - \zeta_M)} = \exp[j\, g\, \Delta l\, (\rho + \rho_o \zeta_M) \sin\alpha \frac{tg\alpha}{tg\varphi}] \tag{18}$$

where,

- Q is the discharge of the water stream;
- ζ is the relative volumetric moisture of the debris deposit (ratio of water volume to the solid mass volume);
- ζ_M is the relative volumetric moisture of the debris deposit at the fluidity limit;
- N is the ratio of solid matter discharge to the water discharge; and
- Δl is the length of the part of the stream under consideration.

Equation (18) can be solved with respect to N by any numerical method of iteration. Model (16)-(18) can be also useful for the computation of rainstorm debris flows. The model predicts the movement, deceleration, and cessation of flows.

Some of the problems stated in this paper are considered in more detail in Vinogradov (1977 and 1980).

References

Vinogradov Y.B., 1977. *Glacialnye proryvnye pavodki i selevie potoki.* Leningrad: Gidrometeoizdat.

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