Chapter 17*

RETAINING WALLS

17.1 INTRODUCTION

Retaining walls are structures used to hold backfill and maintain a difference in the elevation of the ground surface. By the mechanics of performance, retaining walls may be classified as:

1. gravity,
2. tieback,
3. driven cantilever, and
4. reinforced earth.

Figure 17.1 illustrates the mechanics of how each type of wall develops the resistance to react against the imposed lateral earth pressure. The classification of walls, with respect to mechanics of the design and the probable behaviour of the construction medium, is presented in Table 17.1. Types 4,5, and 6 are the ones most prevalent in Nepal.

This chapter aims to provide basic concepts relating to the principles and design of some of the common types of retaining walls. Principles of design of gravity, earth reinforcement, and tieback walls with design examples of masonry gravity type have been presented.

The design of a retaining structure consists of two principal parts, the evaluation of loads and pressures that would act on the structure and the design of the structure to withstand those loads and pressures. The live loads on the structure are estimated, either by using specified codes or by estimation based on available data and experience, depending upon the situation and practices. The evaluation of pressures requires an understanding of earth pressure theories. The earth pressure can be estimated, either by empirical methods or by theoretical analysis, using one or several of the existing theories. Design structure, then, is a matter of analysis for external stability and internal stability based on the mechanics and material properties of the structure.

17.2 LATERAL EARTH PRESSURE

Earth pressure can be classified into **at rest, active,** and **passive**. The calculation of magnitude and distribution of lateral earth pressure, at rest between a soil mass and an adjoining structure, is simplified by assuming the condition of **plane strain,** i.e., strains in the longitudinal dimension are assumed to be zero. The rigorous analysis of earth pressure problems is rarely possible. However, it is the failure condition of the soil mass that is of primary interest and, in this context, provided a consideration of displacement is not required, it is possible to use the concept of plastic collapse. Earth pressure problems can thus be considered as problems in plasticity.

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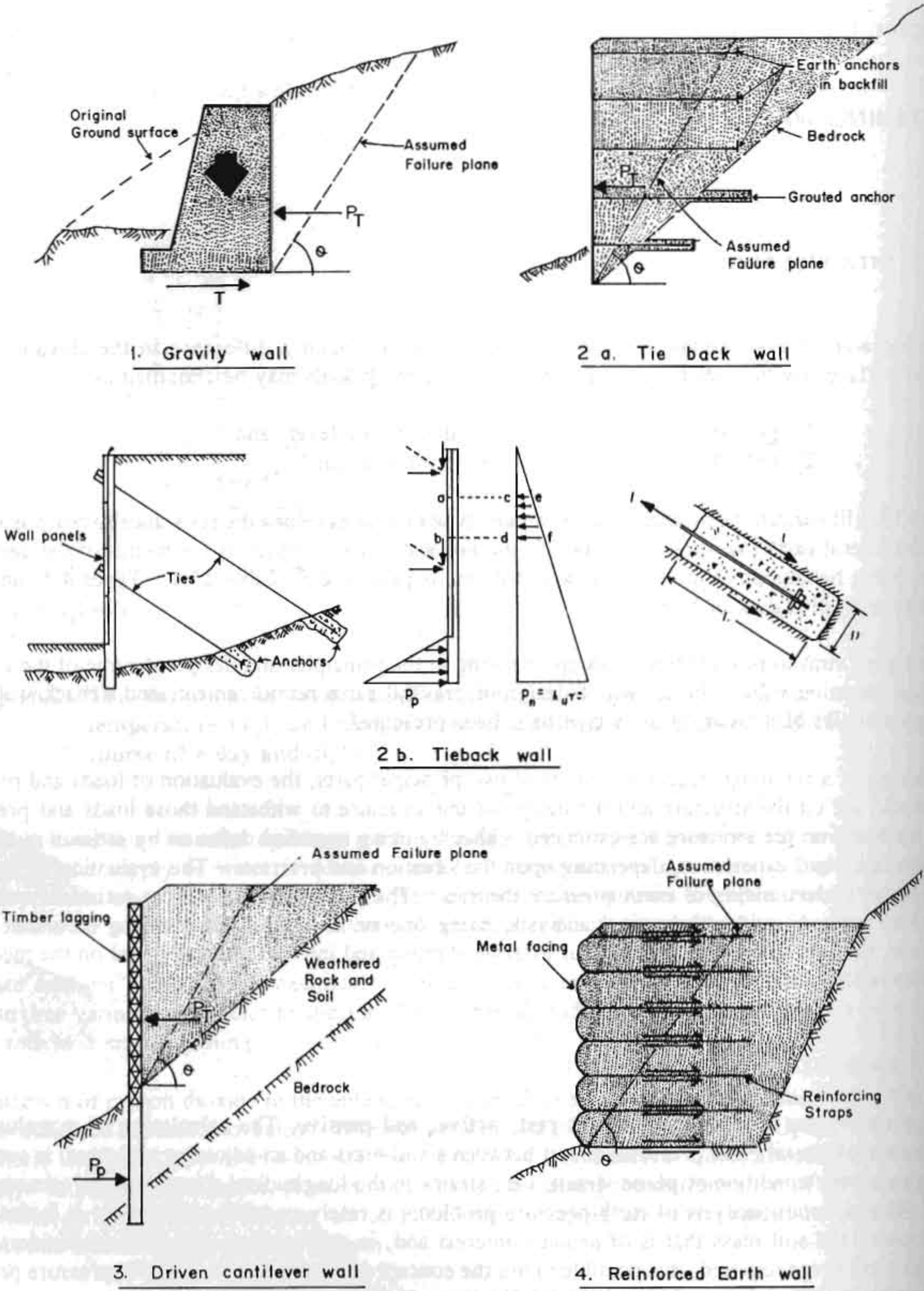
* Tables and Figures for which sources are not credited in this chapter are compilations of the author(s).



Source: Adopted from Driscoll 1979 and Winterkorn and Fang 1975

Fig. 17.1 Mechanics of wall systems

Table 17.1 Wall classification

Gravity	Anchored
1. Bin Walls	1. Vertical Culvert Pipe
	2. Horizontal Sheet Pile
a. Rectangular	3. H-Pile, Timber Lagged
b. Circular	4. Vertical Sheet Pile
c. Cross Tied	5. Stack Sack
	6. All Gravity Structures
2. Concrete Crib	7. Ter-Voile Structure
3. Timber Crib	
4. Gabions	**R.C.C. Walls**
5. Cement Masonry	
5. Dry Stone Masonry	1. Cantilever
6. Concrete Cantilever	2. L Type
7. Drum Walls	3. Buttressed Wall (front or back)
	4. Frame Retaining Wall
Reinforced Backfill	
	Cantilever Piles
1. Reinforced Earth	1. Vertical Sheet Piles
2. Fabric	2. H-Pile, Timber Lagged
3. Stack Sack	

Plastic collapse occurs after the state of plastic equilibrium has been reached in part of a soil mass. Plastic equilibrium is said to be the state at which the shear stress, at every point within the soil mass, reaches the state of yield stress represented by point Y in Figure 17.2 which is an idealized stress-strain relationship in a soil mass. The use of this relationship implies that soil after yielding behaves as a perfectly plastic material, with unrestricted plastic flow taking place at constant stress, and that yielding and shear failure both occur at the same time.

The applied load system, including body forces, at plastic collapse is referred to as the **collapse load.** Determination of collapse load using the plasticity theory is complex. However, complex analysis can be avoided by using limit theorems of plasticity to determine lower and upper bounds to the true collapse load. The limit theorems can be stated as follows.

The Lowerbound Theorem states that if a state of stress can be found that at no point exceeds the failure criterion for the soil and which is in equilibrium with a system of external loads (which includes self weight of soil), then collapse cannot occur: the external load system thus constitutes a lower bound to the true collapse load.

The Upperbound Theorem states that if a mechanism of plastic collapse is postulated and if, in an increment of displacement, the rate of work done by a system of external loads is equal to the rate of dissipation of energy by the internal stresses, collapse must occur: the external load system thus constitutes an upper bound to the true collapse load.

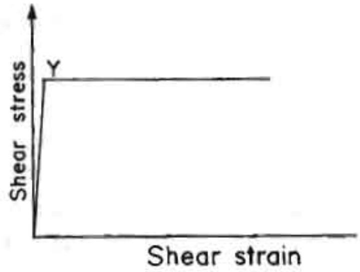
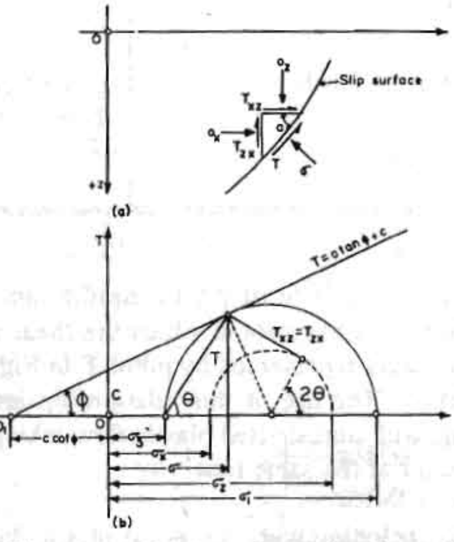


Fig. 17.2 Idealized stress-strain relationship

17.2.1 *Equations for Static Conditions for Stresses in a Two-dimensional Case*

The plastic state of stress distribution is characterized by a slip surface such as the one shown in Figure 17.3a. The Mohr's circle representing the state of stress at any point touches the failure envelope which is a Mohr-Coulomb straight line (Fig. 17.3b).



Source: Winterkorn and Fang 1975

Fig. 17.3 (a) Stresses acting on elements along the sliding surface
(b) Failure condition and Mohr's circle.

From Figure 17.3 we can write the following equations for static conditions:

$$\frac{\partial F}{\partial s} = 0 \tag{17.1}$$

where,

$$F = (\tau - \sigma \tan \phi) \tag{17.2}$$

must be valid on the surface of sliding.

The following are the three basic equations that express in mathematical form the static conditions for stresses in the two-dimensional case $(\sigma_x, \sigma_z, \tau_{xz} = \tau_{xz})$:

$$\frac{\partial\, \sigma_x}{\partial\, x} + \frac{\partial\, \tau_{xz}}{\partial\, z} = 0$$

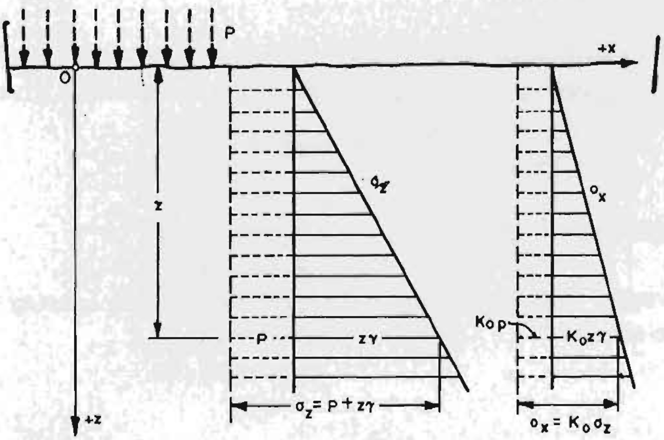
$$\frac{\partial\, \sigma_x}{\partial\, z} + \frac{\partial\, \tau_{zx}}{\partial\, x} = \gamma$$

$$[\frac{1}{2}\,(\sigma_z + \sigma_x) + c\, \cot\, \phi]\, \sin\phi - [\frac{1}{4}\,(\sigma_x - \sigma_x)^2 + \tau_{xz}{}^2]^{\frac{1}{2}} = 0\ . \qquad (17.3)$$

The material has been characterized by three different constants: the bulk density γ, the coefficient of friction $\tan\phi$, and the cohesion c. These can be either zero or greater than zero.

17.2.2 *Lateral Earth Pressure for At-rest Condition*

Figure 17.4 represents the cross section of a halfspace bounded by a horizontal surface. The halfspace is either unloaded or loaded with a uniformly distributed load. The stresses are independent of x; the lines z=const and x=const are principal stress directions σ_z and σ_x are principal stresses, and $\tau_{xz} = 0$.



Source: Winterkorn and Fang 1975

Fig. 17.4 Semi-infinite half space: vertical and horizontal stresses in the at rest condition

Thus, from Equation 17.3

$$\frac{\partial\ \sigma_z}{\partial\ z} = \frac{d\ \sigma_x}{d\ z} = \gamma$$

and

$$\sigma_z = z\gamma \quad .$$

From Figure 17.4

$$\sigma_z = z\gamma + p \quad . \tag{17.4}$$

Experience and experiments show that in the state of complete rest (at rest condition):

$$\frac{\sigma_x}{\sigma_z} = const = K_o \tag{17.5}$$

and

$$\sigma_z = \gamma z + p$$

$$\sigma_x = K_o\ z\ \gamma + K_o\ p$$

for

$$p = 0,\ \sigma_x = K_o\ Z\ \gamma \quad . \tag{17.5 a}$$

The stress distribution is thus hydrostatic (Fig. 17.4). The quantity K_a is the coefficient of earth pressure at rest; numerical values are given in Table 17.2. A good approximation is given by:

$$K_o = l - \sin\phi \tag{17.6}$$

which fits most of the experimental data.

In this case the soil is prevented from expanding or compressing laterally ($\epsilon x = \epsilon y = 0$) and from the theory of elasticity we obtain the equation:

$$\sigma_x = \frac{\nu}{l - \nu}\ \sigma_x \tag{17.7}$$

where **v** is Poisson's Ratio.

Earth pressure acting on walls that do not yield laterally - e.g., stiff, heavy walls, caissons, rigid frames, etc. - can be computed from the following equation (Fig. 17.5):

$$E_o = K_o \frac{h^2\gamma}{2} + K_o ph \tag{17.8}$$

$$E_o = K_o \frac{h^2\gamma}{2} \qquad \text{for } p=0.$$

For a slanting wall (Fig. 17.5b) with no surcharge:

$$E_o = \frac{h^2\gamma}{2} \sqrt{K^2_o + \tan^2 \beta} \tag{17.9}$$

and

$$\tan\delta = \frac{(l-K_o) \cot\beta}{l+K_o \cot^2\beta} \quad . \tag{17.10}$$

Equation 17.5 is valid for effective stresses only. Therefore, if there is groundwater present, but no water movement occurs, the neutral stresses must be considered separately:

$$\sigma_z = \overline{\sigma_z} + u$$

and

$$\sigma_x = K_o \overline{\sigma_z} + \mu = K_o \sigma_z + (1-K_o) u \quad .$$

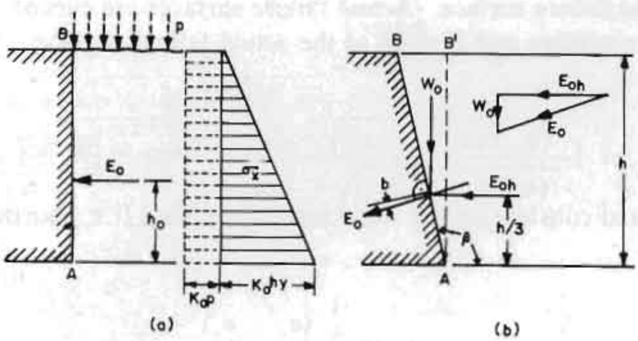


Fig. 17.5 **Earth pressure at rest acting on (a) a vertical and (b) an oblique surface**

Table 17.2 Coefficient of earth pressure at rest

Soil	W_l	I_P	K_a
Loose sand, saturated	-	-	0.46
Dense sand, saturated	-	-	0.36
Dense sand, dry (e=0.6)	-	-	0.49
Loose sand, dry (e=0.8)	-	-	0.64
Compacted, residual clay	-	9	0.42
Compacted, residual clay	-	31	0.66
Organic silty clay, undisturbed	74	45	0.57
Kaolin, undisturbed	61	23	0.64-0.70
Sea clay (Oslo), undisturbed	37	16	0.48
Quick clay	34	10	0.52

Source: Bishop 1957, 1958, Bernatzils 1947, and Simons 1958.

17.2.3 *Active and Passive Earth Pressure*

Active earth pressure is associated with the movement of the wall away from the backfill and passive pressure is associated with the wall moving into the backfill. In designs of earth pressure for retaining walls, with no external forces acting on the backfill, active pressure from the backfill is the only required lateral earth pressure. The passive resistance, due to earth filling or a small embedment in front of the wall, if any, is normally neglected since these cannot be relied upon unless it is certain that the soil in front of the wall will not be eroded.

Active earth pressure can be determined by using Rankine's Theory, Coulomb's Wedge Theory, or Log Spiral Theory. Rankine's Theory (1857) may also be interpreted as a lower bound case of plastic failure. It ignores friction between wall and backfill. Coulomb's Wedge Theory may also be interpreted as a lower bound case of plastic failure and accounts for wall friction as well. The Rankine and Coulomb theories both assume a plane failure surface. Actual failure surfaces are curved. The Log Spiral Method considers the curved failure surface and is close to the actual failure surface.

The Rankine Active State

Referring to Figure 17.3, and considering the Rankine Active State (i.e., horizontal stress equals active pressure):

$$\sin \phi = \frac{\frac{1}{2}(\sigma_1 - \sigma_3)}{\frac{1}{2}(\sigma_1 + \sigma_3 + 2\, c \cot\phi)}$$

$$\sigma_3 = \sigma_1 \left(\frac{1 - \sin\phi}{1 + \sin\phi}\right) - 2c \sqrt{\left(\frac{1 - \sin\phi}{1 + \sin\phi}\right)}$$

$$= \sigma_1 \tan^2 \left(45^\circ - \frac{\phi}{2}\right) - 2\, c \tan \left(45^\circ - \frac{\phi}{2}\right) \quad .$$

For no surcharge, i.e., p=0:

$$\sigma_1 = \gamma z$$

and if,

$$\frac{1 - \sin\phi}{1 + \sin\phi} = K_A$$

$$\sigma_3 = K_A \gamma z - 2c\sqrt{K_A} \quad . \tag{17.11}$$

For c = 0:

$$\sigma_3 = K_A \gamma z \quad .$$

Now, referring to Figure 17.6:

$$P_a = \frac{1}{2}\, \gamma\, H^2\, K_a - cH \sqrt{K_a} \; . \tag{17.12}$$

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} \tag{17.13}$$

$$K_a = \cos\beta \left[\frac{\cos\beta - \sqrt{(\cos^2\beta - \cos^2\phi)}}{\cos\beta + \sqrt{(\cos^2\beta - \cos^2\phi)}}\right] \quad . \tag{17.14}$$

For non-cohesive backfill:

$$P_a = \frac{1}{2}\, \gamma\, H^2\, K_a \quad .$$

The basic assumptions in Rankine's Active State Theory are that:

1. the soil is homogeneous and isotropic, possesses internal friction, and is in a state of plastic equilibrium,

2. the failure surface within the backfill is planar,

3. the shear strength is mobilized uniformly on all planes throughout the backfill,

4. the presence of the wall does not influence the state of stress in the backfill,
   - the failure is a two-dimensional problem, and
   - the resultant, Pa, is inclined at angle ß to the wall.

Figure 17.6 shows soil structure system and force polygons for Rankine's Active State

Coulomb's Theory

Coulomb's Theory (1776) involves consideration of the stability, as a whole, of the wedge of cohesionless soil between a retaining wall and a trial failure plane. Friction between the wall and the adjacent soil is taken into account. This friction angle is denoted by σ and can be determined in the laboratory by means of a direct shear test. A number of trial failure planes would have to be selected to obtain the maximum value of P, corresponding to a particular value of Θ, and is given by:

$$\frac{\partial P}{\partial \theta} = 0 \quad .$$

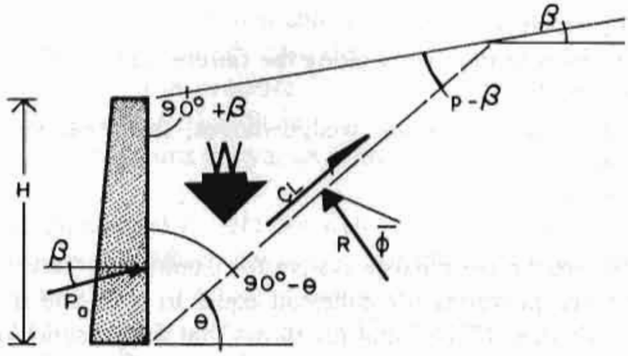
This leads to the following solution for P, (see Figure 17.7):

$$P_{a} = \frac{1}{2} K_{A} \gamma H^{2} \qquad \text{for } c = 0 \text{ and } Q = 0$$

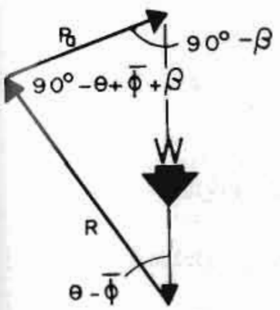
where,

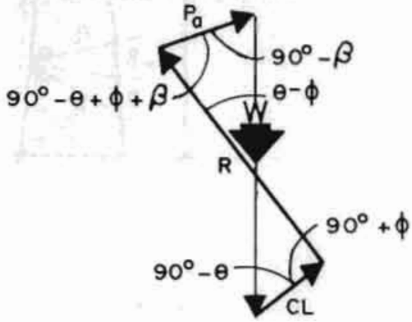
$$K_{A} = \left(\frac{\dfrac{\sin(\alpha+\phi)}{\sin\alpha}}{\sqrt{[\sin(\alpha-\sigma)]} + \sqrt{\left[\dfrac{\sin(\phi+\sigma)\,\sin(\phi-\beta)}{\sin(\alpha+\beta)}\right]}} \right)^{2} \quad . \tag{17.15}$$

The point of application of total thrust is not given by Coulomb's Theory but is assumed to act at a distance of H/3 above the base of the wall in the case of dry backfill. In the case of partly submerged backfill, γ will be equal to submerged unit weight in the submerged part.



(a) Soil structure system

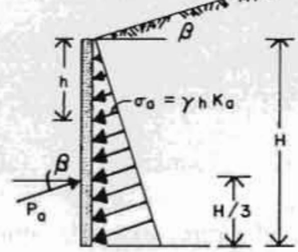




(b) Force polygon for non-cohesive backfill

(c) Force polygon for cohesive backfill

RANKINE ACTIVE CASE



(d) Pressure diagram

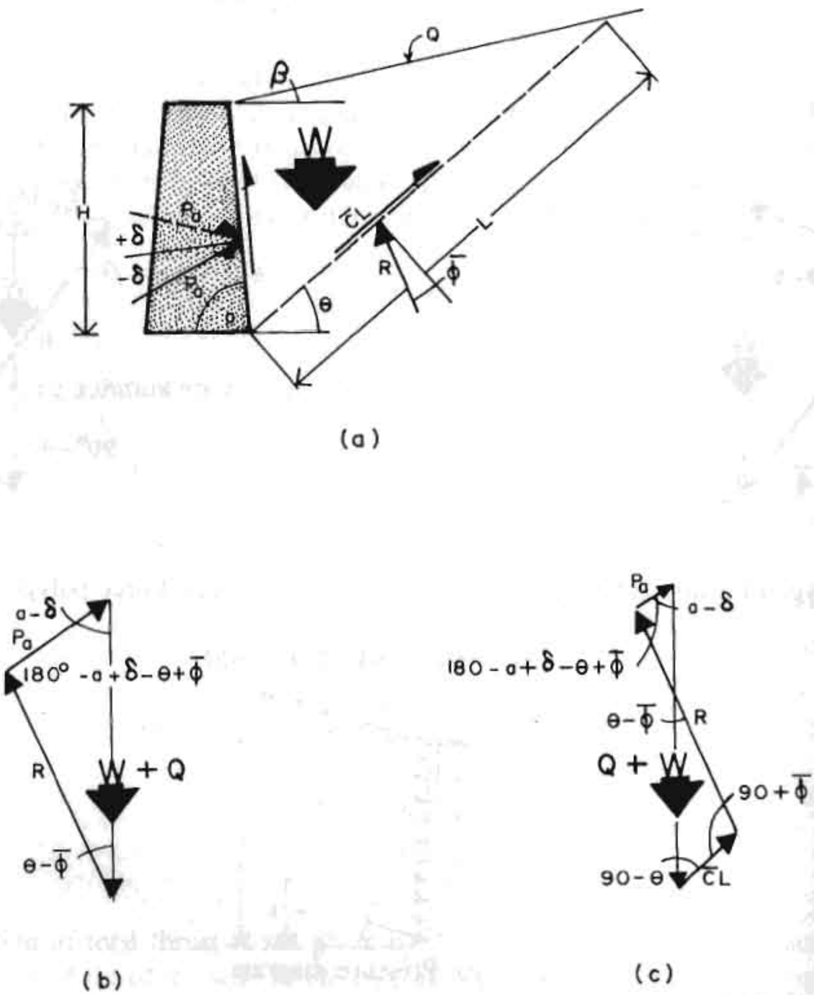
Source: Driscoll 1979

Fig. 17.6 Earth pressure diagram, Rankine Active Case for dry backfill

The basic assumptions in the Coulomb analysis are that:

1. the soil is homogeneous and isotropic, possesses internal friction and cohesion, and is in a state of plastic equilibrium,
2. the failure surface in the backfill is planar,
3. the shear strength, τ, is mobilized uniformly along the failure plane, and $\tau = \overline{c} + \overline{\sigma_n} \tan\phi'$,
4. the failure wedge is a rigid body.
5. there is wall friction; that is, as the failure wedge moves, the shear strength along the soil-wall interface is mobilized, and
6. the failure is a two-dimensional problem.

Figure 17.7(a) shows the soil-structure-surcharge system for Coulomb's Active Case. Figures 17.7f(b) and (c) show the resulting force polygons for cohesion equal to zero and a value greater than zero, respectively. Comparison of Figures 17.7(b) and (c) shows that the presence of cohesion decreases the value of P_a; the total active thrust. Equation 17.12 has also been generalized for seismic condition and surcharge as in Equations 17.14 and 17.17.



Source: Driscoll 1979

Fig. 17.7 Coulomb's Active Case

17.3 RIGOROUS DESIGN OF RETAINING WALLS

Retaining walls and breast walls may be designed using semi-empirical methods or theoretical methods employing Rankine's Active State or Coulomb's Theory or the Log Spiral Theory for earth pressure calculations. Graphical methods are also available for determination of earth pressures by theoretical methods. The Log Spiral Theory is generally considered to be the most accurate of these three methods. However, the variation in active earth pressure from Coulomb's Theory is about 10 per cent on the conservative side, while Rankine's Active State results in about 10 per cent variation on the non-conservative side. For passive pressure the values from these three theories are widely divergent.

This section presents a design method for gravity walls employing Coulomb's Wedge Theory. Design examples by manual and computer methods have been presented. A computer programme is very helpful in developing various designs for the selection of the most economical option. Section 17.3.2 and 17.3.3 provide concepts of design for a Reinforced Cement Concrete (R.C.C.). Crib Wall and Tieback Wall. The empirical design of an Earth Reinforced Wall is presented in Section 17.3.4.

17.3.1 *Design of Gravity Type Retaining Wall*

All gravity walls (both retaining walls and breast walls) are designed as simple rigid walls. For purposes of analysis, equivalent sections of a gabion wall are considered as shown in Figure 17.8.

a) *Design Criteria*

i)	Factor of safety against overturning (F_o)	> 2.0 (static loads)
	Factor of safety against overturning (f_o)	> 1.5 (earthquake forces)
ii)	Factor of safety against sliding (F_s)	> 1.5 (static forces)
	Factor of safety against sliding (f_s)	> 1.0 (earthquake forces)
iii)	Highest base pressure, f_{max} (static)	$< q_a$ (allowable bearing capacity)
	Lowest base pressure, f_{min} (static or dynamic)	> 0
	Highest base pressure (dynamic), f_{max} (dynamic)	< 1.25 qa

iv) If the hill slope itself is not stable, the wall cannot be stable, as the cylindrical slip surface may pass below the toe and heel of the wall. So stability of the slope should also be checked (Fig. 17.8). Indirectly the criteria in (iii) account for an unstable slope as the allowable bearing pressure will be very low in that case.

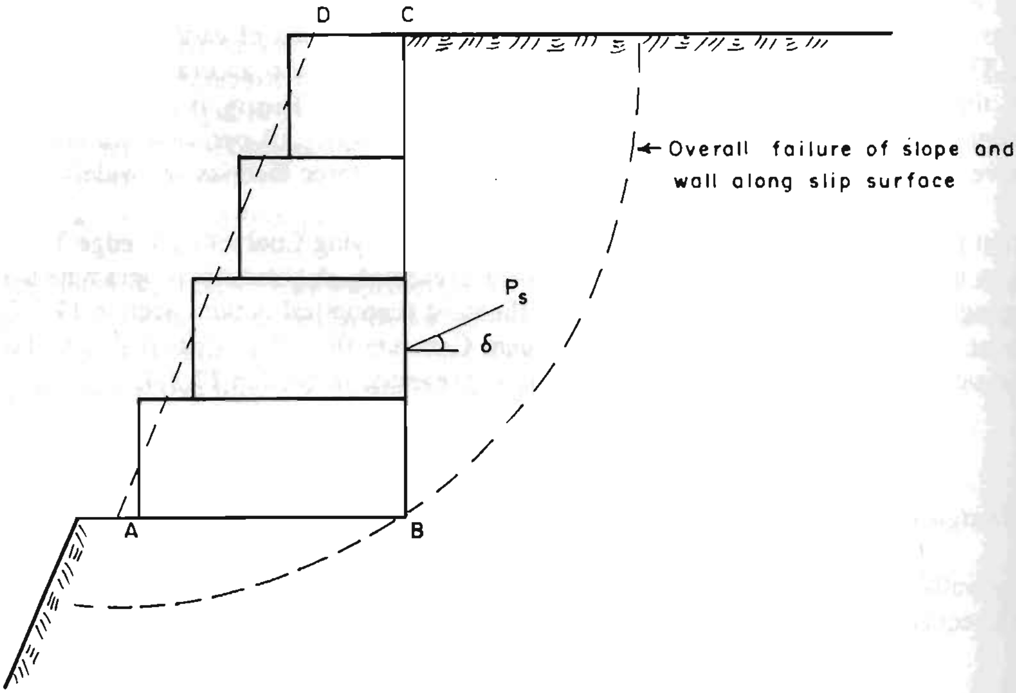


Fig.17.8 Assumed trapezoidal section of gabion wall (ABCD)

For low volume roads, it is suggested that walls may not be designed for earthquake forces. Otherwise, wall sections will become too thick and uneconomical. It would be more economical to repair failed walls after earthquakes. It may also be mentioned here that the significant tilt of the base of the wall towards the hillside will ensure its dynamic stability against earthquake forces.

b) *Assumptions*

i) The gravity wall is a rigid body and behaves as one integral body.
ii) Backfill is isotropic and homogeneous material.
iii) Backfill obeys Coulomb's Law of Shear Strength.

$$\tau = c + (\sigma - u) \tan\phi$$

where,

σ = normal stress across failure plane,
u = pore-water pressure,
c = cohesion, and
ϕ = angle of internal friction.

iv) Pore-water pressure u is,

$$u = (\bar{B})\,\sigma$$

$\bar{B}$ = pore pressure coefficient
0 for dry backfill, and
0.1 to 0.4 for moderate to heavy seepage pressures.

v) Earth pressure (both static and dynamic) acts at an angle of σ with normal to the back face of the wall (Fig. 17.9). The angle of wall friction(s) is generally assumed as to be:

$$\delta = \frac{2\phi}{3} \quad .$$

Adhesion is neglected on the back face of the wall.

vi) Static earth pressure (P_s) acts at one third the height of the wall. Whereas the dynamic increment of earth pressure **(Pdi)** acts at half the height of the wall (Fig. 17.9).

vii) The shearing resistance (τ) at the interface of the base of the wall and the foundation is given by:

$$\tau = Ca + \mu\ (\sigma - u)$$

where,
Ca = adhesion, and
μ = coefficient of friction (= tanσ)

viii) The slip surface in backfill is plane and inclined at an angle of Θ with the horizontal. It is assumed that the wall will move towards the valley and the wedge of the backfill will tend to slide on the critical slip surface. A critical slip surface is that slip surface which gives maximum earth pressure. Critical slip surfaces are different for static and dynamic conditions. This results in conservative estimates of earth pressures.

ix) There is no progressive failure of backfill and strength is uniformly mobilized along the slip surface.

Actually if a displacement analysis of wall-backfill interaction is made, earth pressure will not be found to vary linearly with height. In actual practice, a theoretical slip surface may not develop at all due to the steep excavation of the hill profile. The present design method, based on the above assumption, is a little conservative.

c) *Design Parameters*

Values of backfill parameters, and properties of structural materials and foundation may be obtained from Chapter 9 of the manual or from any standard text or handbook. A laboratory check is sometimes justified when the size of structure and economies from design are considerable.

d) *Forces on Retaining Wall*

Forces on retaining wall (a, b, c, d) and the wedge (b, d, e) are shown in Figure 17.9. Details are given below.

Forces on Wedge

i)	Weight of Wedge	=	W_w
ii)	Weight due to vertical earthquake acceleration	=	$\alpha v W_w$
iii)	Force due to horizontal earthquake acceleration	=	$\alpha_h W_w$
iv)	Reaction at an angle of the normal on slip surface	=	R
v)	Surcharge on backfill	=	Q_s
vi)	Surcharge due to vertical component of earthquake	=	$\alpha_v Q_s$
vii)	Force due to horizontal earthquake acceleration on surcharge	=	$\alpha_h Q_s$
viii)	Uplift due to pore pressure on slip surface	=	U
ix)	Static earth pressure	=	P_s
x)	Dynamic increment of earth pressure	=	Pdi
xi)	Cohesion along slip surface = c x de	=	C

Forces on Wall

i)	Weight of retaining wall	=	W_R
ii)	Weight due to vertical earthquake acceleration	=	$\alpha v W_R$
iii)	Force due to horizontal earthquake acceleration	=	$\alpha h W_R$
iv)	Base friction	=	F
v)	Adhesion at base = Cax base width	=	Cab_2
vi)	Normal reaction at base	=	V_f
vii)	Uplift on base of wall	=	V

It may be seen that a_3 in Figure 17.9 will be a negative quantity for breast walls or retaining walls with negative batter.

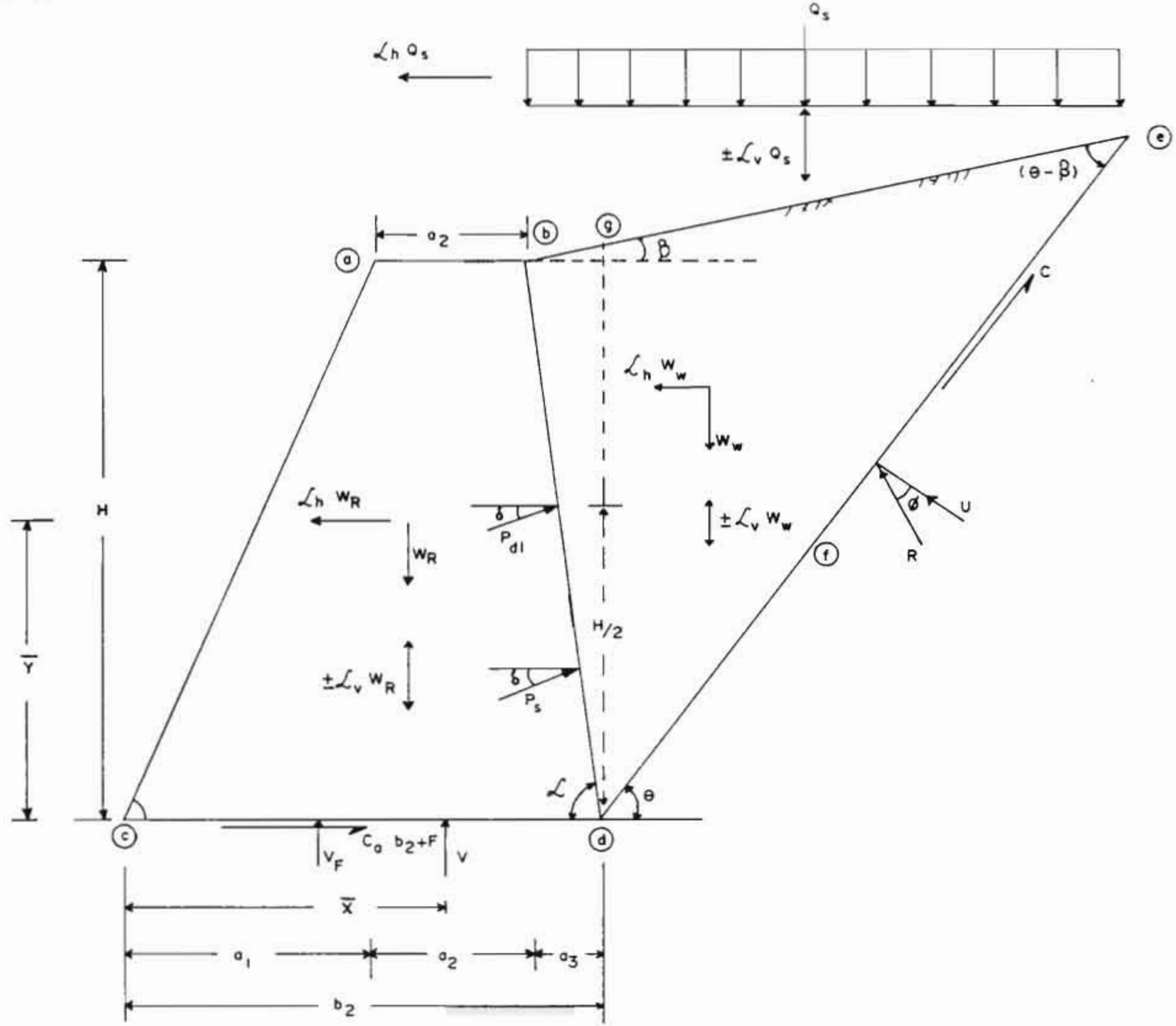


Fig. 17.9 Forces on retaining walls

e) *Earth Pressure on Wall*

The total pressure Pd(=P_s + Pdi) is obtained by the following expression after considering the equilibrium of the wedge (Viladkar et al. 1985):

$$P_d = \frac{(W_w + Q_s)\ [\alpha_h \cot(\theta-\phi) + 1 \pm \alpha_v] - U\ [\cos\theta - \cot(\theta-\phi)\sin\theta] - c\ [\sin\theta + \cot(\theta-\phi)\ \cos\theta]}{\cos(\alpha-\delta)\ [1+\tan(\alpha-\delta)\ \cot(\theta-\phi)]} \tag{17.13}$$

$$= \frac{1}{2}\ \gamma_s\ H^2\ K_A + H\ K_A\ \sin\alpha\ \mathrm{cosec}(\alpha+\beta)\ Q_s\ . \tag{17.14}$$

For

$$\alpha_h,\ \alpha_v = 0$$

$$P_d = P_s + P_{di} = P_s + 0 = P_s \tag{17.16}$$

where,

$$U = 0.5\ \bar{B}\ \gamma_s\ [H\ (\mathrm{cosec}\alpha\ \ \sin(\alpha+\beta)]^2\ \sec\beta\ \mathrm{cosec}(\theta-\beta) \tag{17.17}$$

$$C = cH\ \mathrm{cosec}\alpha\ \sin(\alpha+\beta)\ \mathrm{cosec}(\theta-\beta) \tag{17.18}$$

where,

γ_s = unit weight of backfill or soil,
α = angle between back face of wall and its base,
Θ = dip of slip surface,
ß = slope of backfill, and
H = height of wall above heel.

For cohesionless backfill, the earth pressure coefficient K_A is given by:

$$K_A = (\ 1 \pm \alpha_v\ )\ \sin^2(\alpha-\lambda+\phi)\ [\cos\lambda\ \sin^2(\alpha-\lambda)\ \sin(\alpha-\lambda-\delta)\ (1+\frac{1}{\sqrt{m}})^2] \tag{17.19}$$

where,

$$m = \frac{\sin(\phi+\delta)\ \sin(\phi-\beta-\lambda)}{[\sin(\alpha+\beta)\ \sin(\alpha-\lambda-\delta)]} \tag{17.20}$$

$$\lambda = \tan^{-1}\left[\frac{\alpha_h}{1\pm\alpha_v}\right] \ . \tag{17.21}$$

f) *Factors of Safety*

The dynamic factor of safety against sliding is:

$$F_s = \frac{\mu\ [-\nu + W_R\,(1\pm\alpha_v) + Pd\ \cos(\alpha-\delta)] + Cab_2}{[\alpha_h\ W_R + Pd\ \sin(\alpha-\delta)]} \tag{17.22}$$

$$= F_s\ (static),\ \alpha_v = \alpha_h = 0$$

where,

$$u = 0.5\ \gamma_s\ \overline{B}H\ \mathrm{cosec}\alpha\ \sec\beta\ \sin(\alpha+\beta)\ b_2\ . \tag{17.23}$$

The factor of safety against overturning is:

$$F_o = \frac{(1\pm\alpha_v)\ W_R\ \bar{x} + \cos(\alpha-\delta)\ [P_s\ (\alpha_1+\alpha_2) + (\alpha_3 - \frac{H\cot\alpha}{3}) + Pdi\ \{\alpha_1 + \alpha_2 + \alpha_3 - \frac{H\cot\alpha}{2})\}]}{\alpha_h\ W_R\ \bar{y} + H\ \sin(\alpha-\delta)\ [\frac{P_s}{3} + \frac{Pdi}{2}] + 2b_2\ \frac{V}{3}} \tag{17.24}$$

$$= F_o\ (static),\ \alpha_h = \alpha_v = 0$$

where,

$\bar{x}$, $\bar{y}$ = coordinate of centre of gravity of wall with respect to its toe (Fig. 17.9).

g) *Base Pressures*

The resultant of the line of action of all forces will strike the base of the wall at a distance of $\bar{x}_1$, from the toe as follows:

$$\bar{x}_1 = \frac{W_R\,(1 \pm \alpha v)\,\bar{x} - \alpha_h\,W_R\,\bar{y} + P_s\,[\cos(\alpha-\delta)\,(b_2 - \frac{H\cot\alpha}{3}) - H\,\frac{\sin(\alpha-\delta)}{3} + M_d]}{W_R\,(1 \pm \alpha_v) + Pd\,\cos\,(\alpha-\delta)} \;. \qquad (17.25)$$

where,

$$Md = Pdi\,[\cos(\alpha-\delta)\,(b_2 - \frac{H\cot\alpha}{2}) - \frac{H\sin(\alpha-\delta)}{2}] \;.$$

Thus the eccentricity (**e**) of the resultant force is given by:

$$e = \frac{b_2}{2} - \bar{x} \;. \qquad (17.26)$$

The dynamic base pressures are:

at toe

$$f_1 = \frac{V_F}{b_2}\,(1 + \frac{6_e}{b_2}) \qquad (17.27)$$

at heel

$$f_2 = \frac{V_F}{b_2}\,(1 - \frac{6e}{b_2}) - 2\frac{v}{b_2} \qquad (17.28)$$

The static analysis is obtained by substituting α_h and α_v equal to zero in the above equations. The analysis is solved for different wall dimensions until design criteria are satisfied. Thus the final wall section is arrived at. A typical example is solved for cohesionless backfill and for a drystone masonry wall and is presented in the following pages.

h) *Computer Aided Design of Retaining Walls*

To save time from calculations and iterations, a computer programme RETAIN has been developed. Using Equation 17.16, the programme first computes the critical wedge angle Θ, following an iterative procedure for which the static earth pressure P_s is maximum. The static factors of safety for sliding and overturning are calculated from Equations 17.22 and 17.24 for initially assumed wall dimensions. Static base pressures are also checked from Equations 17.27 and 17.28. If all the design criteria are not satisfied, wall dimensions are changed and all of the above calculations are repeated. Iterations are made until the desired wall dimensions are obtained. Similarly calculations are repeated for earthquake forces. The programme thus gives wall dimensions for a dynamic case also after satisfying all the design criteria for the dynamic case.

The RETAIN programme has been used for the last 15 years to design many retaining walls. Realistic and economical wall sections are easily obtained for static conditions. For dynamic conditions, the programme gives too large a section for a low volume hill road if the base of the wall is not tilted. Figure 17.10 gives some computer results. The design parameters are:

$$c = 0;\ \phi = 30°,\ \beta = 0,\ Y\ wall = 1.9\ t/m^2,$$

$$Y\ soil = 2.0\ t/m^2,\ \beta = 0,\ Ca = 0,\ q_a = 20\ t/m^2,\ q_s = 2\ t/m^2$$

$$\alpha_h = 0,\ \alpha_v = 0,\ H = 6m\ .$$

Figure 17.10 clearly shows that a negative batter of as small as 5:1 can result in substantial savings in the cost of a wall. In the case of a wall with a negative batter of 3:1, the saving is even more. This concept became clear only after using the above programme.

i) *Example of Retaining Wall Design by Manual Calculations*

<u>Typical Design Example</u>

Given H = 8m, ϕ = 40°, δ = 22.5°, μ = 0.6, α_h = 0.08, γ_s= 1.8 t/m³, and γ_m= 2.0 t/m³, design a masonry retaining wall section when the bearing capacity of the rock under normal loads is 15t/m². Forces acting on the wall are shown in Figure 17.11

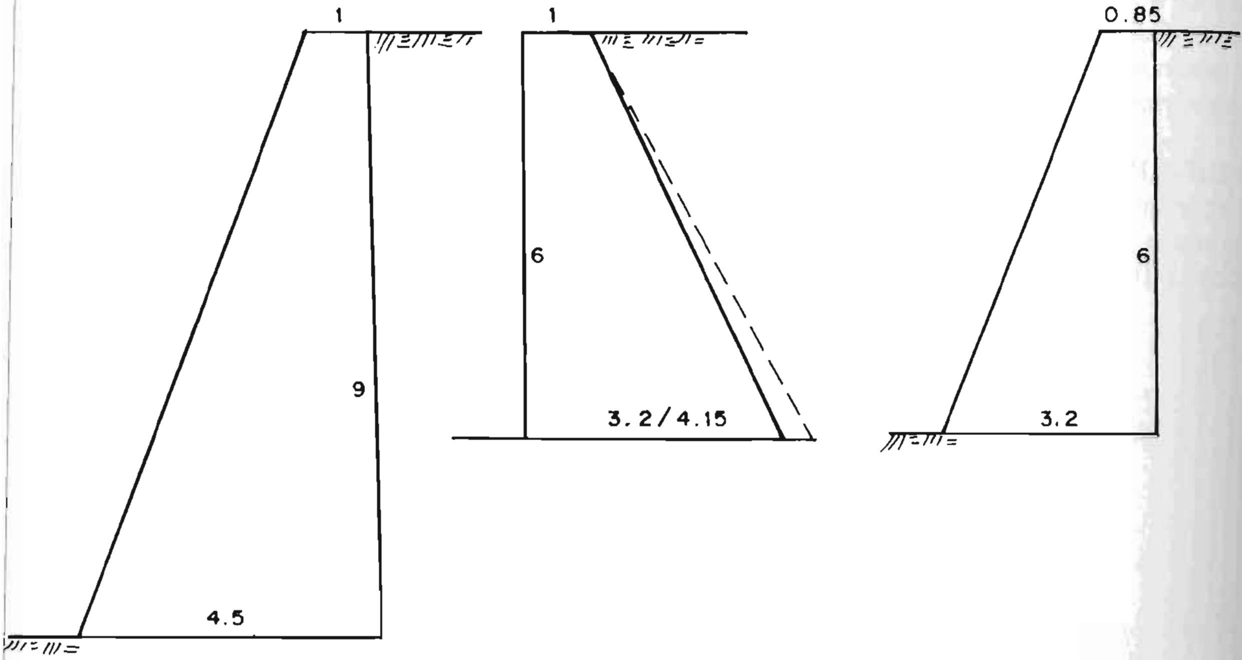
The retaining wall section is adopted, as having a top width **b** equal to 0.6m, face slope 1 horizontal to 3 vertical, base slope 6H:1V and the toe projection as 0.45m wide and 0.675m high as shown in Figure 17.12.

For ϕ=40°, σ=22.5°, and α_h=0.08, α=90°, ß=0; from Equation 17.19,

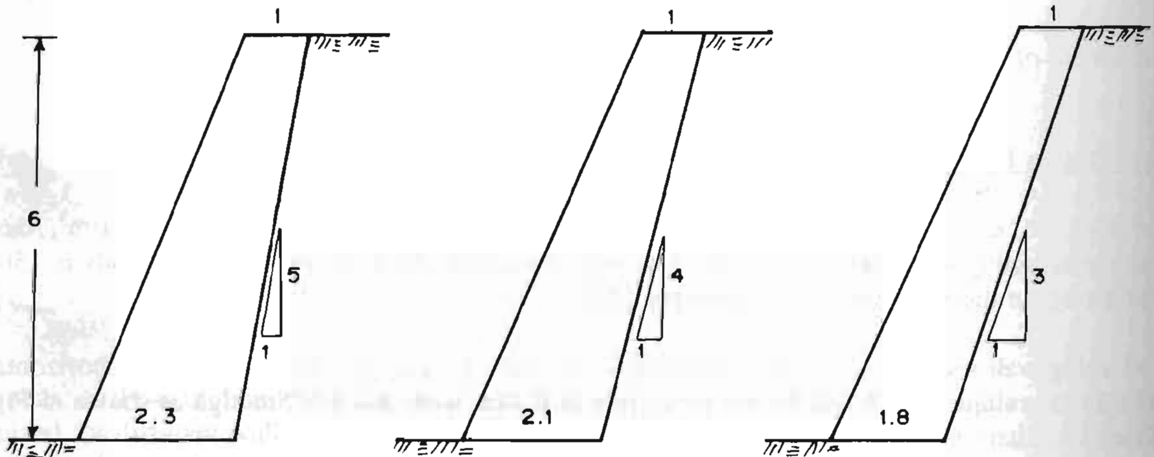
$$Ka = 0.1992,\ Kad = 0.2485.$$

Static earth pressure:

$$Ps = (1/2)\ \gamma_s\ KaH^2 = 1/2 \times 1.8 \times 0.1992 \times 8^2$$
$$= 11.50\ t\ \text{acting at (H/3) 2.667m above base.}$$



(a) Positively battered walls



(b) Negatively battered walls

Fig. 17.10 Wall sections (metres) by computer programme–negative battered vs positively battered walls (solid and dotted lines for 40 and 20 t/m² bearing pressures, q_a)

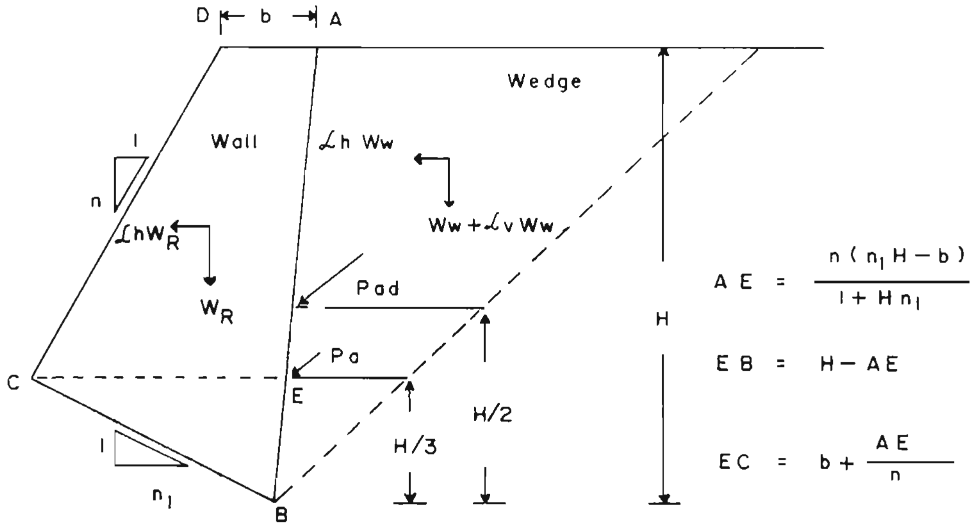


Fig. 17.11 Seismic forces on retaining wall

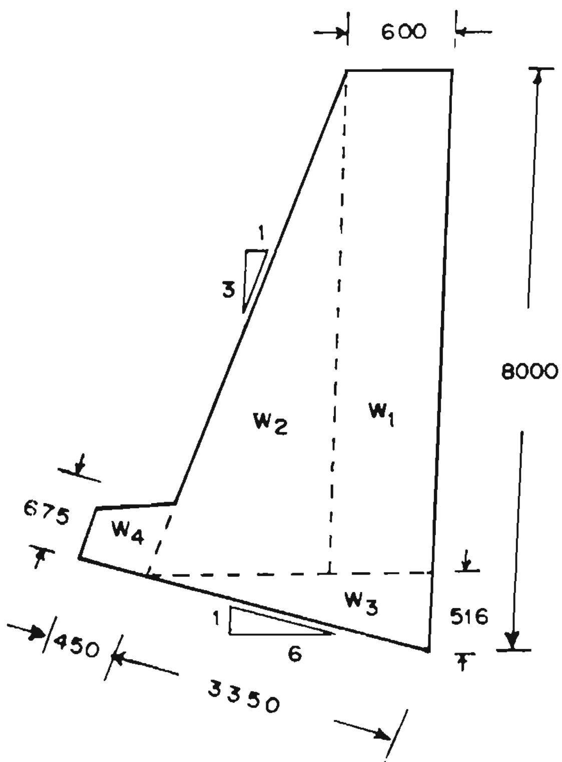


Fig. 17.12 Assumed section

Total dynamic earth pressure:

$$Pd = (1/2)\ \gamma_s\ Kad\ H^2 = 14.26\ t.$$

Dynamic increment:

Pdi = Pd-Ps = 14.26 - 11.50
= 2.76 t acting at (H/2), 4.0m above base.

Both the pressures Ps and Pdi will be inclined at angle 22.5° to the horizontal, where cos 22.5° and sin 22.5° = 0.9239 and sin 22.5° = 0.3827.

From the expressions shown in Figure 17.11 the various dimensions of the present retaining wall shown in Figure 17.12 will be as follows:

$$AE = \frac{n\ (n_1 H - b)}{(1 + H n_1)}$$

$$= \frac{3\ (6 \times 8 - 0.6)}{(1 + 3 \times 6)}$$

$$= 7.484\ m\quad .$$

EB = H-AE = 8.0 - 7.484 = 0.516m
EC = b+AE/n = 0.60 + 7.484/3 = 3.095m

The components of forces and their moments about B are given in the next section.

Retaining Wall Calculations - Moments About Heel (B)
(Horizontal force to left + ve: Anticlockwise moment + ve)

Force Moment	Calculation Force	Horizontal Component	L.A. Moment		Vertical Component	Force L.A.	
		(t)	(m)	(t-m)	(t)	(m)	(t-m)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
P_s	11.50x0.9239	10.60	2.667	28.27	-	-	-
	11.20x0.3827	-	-	-	4.40	0	0
W_1	0.6x7.484x2.0	-	-	-	8.98	0.30	2.69
W_2	0.5 (3.095-0.6)x 7.484 x 2.0	-	-	-	18.67	1.432	26.74
W_3	0.5 x 3.095 x 0.516 x 2.0	-	-	-	1.60	1.032	1.65
W_4	0.4 x 0.675 x 2.0	-	-	-	0.61	3.320	2.02
	Sub total	10.60	-	28.27	34.26	-	33.10

1	2	3	4	5	6	7	8
Pdi	2.76 x 0.9239	2.55	4.0	10.20	-	-	-
	2.76 x 0.3827	-	-	-	1.06	0	0
EQ1	0.08 x 8.98	0.72	4.258	3.06	-	-	-
EQ2	0.08 x 18.67	1.49	3.011	4.50	-	-	-
EQ3	0.08 x 1.60	0.13	0.322	0.04	-	-	-
EQ4	0.08 x 0.61	0.05	0.854	0.04	-	-	-
Total		15.54	-	46.11	35.32	-	33.10

Check for Sliding

Here the sliding movement will be up the slope, hence the force required to cause sliding is governed by the force resolved along the plane of sliding.

i) <u>Actuating Force</u>

Static case = Ps cos(σ + angle ECB) = Ps cos (22.5 + 9.46°)
= 11.5 cos (31.96°) = 9.76 tons.

With earthquake = 14.26 cos(31.96°) = 12.10 tons.

ii) <u>Resisting Force</u>

$$F = \frac{\mu\ \cos\theta\ +\ \sin\theta}{\cos(\delta+\theta)\ -\ \mu\sin(\delta+\theta)} \times W$$

$$= \frac{0.6\ \cos\ 9.46°\ +\ \sin\ 9.46°}{\cos\ (31.36)\ -\ 0.6\ \sin\ (31.96°)} \times W$$

$$= \frac{0.76}{0.53} \times W$$

$$= 1.425\ W\ .$$

Here Θ is the dip of the foundation towards the hillside.

Static case F = 1.425 W = 1.425 x 34.26 = 48.82 > 9.76 tons <u>ok</u>.
With earthquake F = 1.425 x 35.32 = 50.33 tons > 12.10 tons <u>ok</u>.

(a) Without Earthquake

Total moment about B = 28.27+ 33.10 = 61.37 tons.

Resultant load normal to base = 36.26 cos9.46° + 10.6 sin 9.46°
= 35.54 tons.

Inclined base width = 3.095 sec 9.46° + 0.45 = 3.800m.

Distance of resultant from B = 61.37 / 35.54 = 1.727m,

e = 1.727 - 3.80/2 = - 0.173m, that is towards B.

$$p = \frac{35.54}{3.8}\ [1 \pm \frac{6 \times 0.173}{3.8}]$$

$$= 11.91,\ 6.79\ t/m^2 < 15\ t/m^2$$

o.k.

(b) With Earthquake

Total moment about B = 46.11 + 33.10
= 79.21 t-m.

Resultant load normal to base = 35.32 cos 9.46° + 15.54 sin 9.46°
= 37.39.

Distance of resultant from B = 79.21 / 37.39 = 2.118m,

e = 2.118 - 1.900 = 0.218m,

$$p = \frac{37.39}{3.8}\ [1 \pm \frac{6 \times 0.218}{3.8}]$$

$$= 13.23\ t/m^2$$

Allowable bearing pressure in earthquake conditions will be 25 per cent more, i.e., 15 x 1.25 = 18.75 t/m^2 - hence safe. It is seen that there is a possibility of slight reduction in the toe protection; 0.3m x 0.45m size could be used.

17.3.2 *Crib Walls*

Crib Walls (Fig. 17.13a) may be built of timber, precast concrete, or steel members. Many types of precast concrete and prefabricated steel cribs are available from manufacturers. The insides of the crib are filled with soil and the whole unit acts as a gravity wall.

In the design of crib walls, stringers are considered to act as simply supported beams that span the space between the ties. The stringers that make up the face are subjected to the pressure of the soil inside the crib, while those that make up the back must resist the difference between the earth pressure exerted by the backfill and that of the soil inside. The earth pressure exerted by the backfill on the wall may be computed in the same manner as the gravity wall. The soil inside the crib is usually placed in well-compacted layers and the inside pressure would be close to the at rest earth pressure given in Table 17.3.

The computation for safety, with respect to overturning and sliding, is the same as for the concrete gravity wall. However, not all the weight of the soil in the crib can be counted on to resist the overturning moment. If the wall tilts forward, the soil inside may not move as an integral part of the wall. Rather, it may move downwards with respect to the wall, and shear stresses would develop between the soil and the wall. These shear stresses act vertically downwards and contribute to the resistance against overturning. The stress may be computed by means of the **Arching Theory** (Terzaghi 1943). If the angle of friction between the soil and wall is σ, the average stress on the wall at depth y is:

$$\sigma = \frac{ab}{2(a+b)} \frac{\gamma K_o}{\tan\delta} \{1-\exp [- \frac{2y K_o(a+b)}{ab} \tan\delta] \}. \qquad (17.29)$$

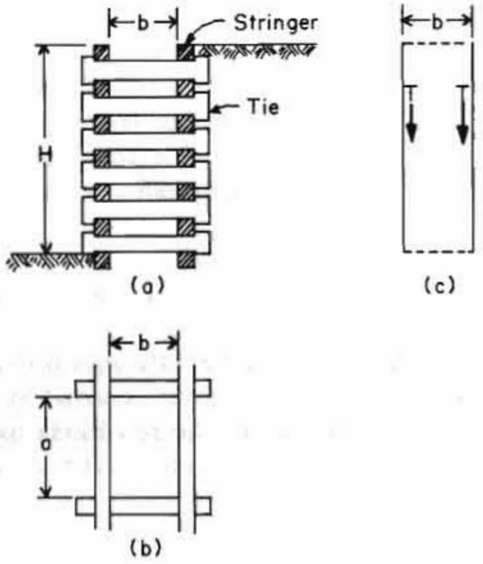
The shear stress between soil and wall is (Fig. 17.14c):

$$T = \sigma_x \tan\delta$$

and the total vertical force transmitted by friction is:

$$F = ab\gamma \{H - \frac{ab}{2(a+b) \tan\delta K_o} [1-\exp (- \frac{2(a+b)}{ab} H \tan\delta K_o)]\} \qquad (17.30)$$

The force F plus the weight of the crib constitutes the weight W to be used for the calculation of resisting forces and moments. The stringers should be designed to resist σ_x. Since stringers are usually of the same size in a given crib wall, the value of y in Equation 17.29 should be set equal to H.



Source: Winterkorn and Fang 1975

Fig. 17.13 Crib walls

Table 17.3 Coefficients of earth pressure at rest

Soil	K°
All types, normally consolidated	1-Sin
Compacted clay, hand tamped	1.0 to 2.0
Compacted clay, machine tamped over entire backfill	2.0 to 6.0
Clay, overconsolidated	1.0 to 4.0
Sand. loosely dumped	0.5
Sand, compacted	1.0 to 1.5

Source: Winterkorn and Fang 1975

17.3.3 *Tieback Wall*

Retaining walls are sometimes supported by tiebacks anchored in firm material as shown in Figure 17.1(2b). Basement walls may be designed to support the sides of excavations during construction. Construction usually begins with the installation of columns, or soldier beams. These are supported by the anchor ties. The wall between the columns may consist of reinforced concrete, precast concrete panels, fabricated metal pieces, or wooden lagging. The ties are stressed to support the columns.

The wall deflection is naturally dependent upon the tension in the tie and may range from almost zero to 7.5cm at the top for walls 15m in height (Shannon and Strazer 1970). If the bottom of the column is not driven into stiff soils, considerable movement may take place at the bottom. Model tests of tieback walls in sand (Hanna and Matallana 1970) have indicated that this may be as large as 0.05H. With such movements, the earth pressure distribution would be close to that for braced excavations. If the bottom of the column is driven into soil and the ties are adequately tensioned, the earth pressure would be close to the at rest earth pressure and the values given in Table 17.3 may be used for design.

Table 17.4 Bond stresses for design of anchors

Soil Type	Bond Stress f
Stiff clay	$5t/m^2$ or 0.25 times unconfined compression strength
Dense sand	$10t/m^2$
Sound rock	$15t/m^2$

Source: Winterkorn and Fang 1975

The wall panels should be designed as beams (Figure 17.1 [2b]) supported by columns. Each panel, such as **ab** in Figure 17.1 (2b), should be designed to support a uniform load equal to the earth pressure in the shaded area **cdef**. If the ties are anchored in sound rock, the columns are designed as continuous beams supported by the ties; and at the bottom. However, a small yield in the anchor can significantly effect the bending moments in the column, in so far as it would reduce the negative moments at the tie and increase the positive moment. If the ties are anchored in soil, the maximum positive moments should be computed on the assumption that the column acts as a simply supported beam between ties (Mansur and Alizadeh 1970). The bottom end of the column is supported by passive earth pressure as shown in Figure 17.1 (2b).

The ties are anchored in rock or firm soil by high strength cement grout. The bond between the grout and the surrounding soil provides the anchor age and must resist the tension in the tie-rod. If T is the tie-rod tension and **f** if the bond Figure 17.1 (2b), then:

$$T = \pi DLf \tag{17.31}$$

where,

D is the diameter of the drilled hole.

In current design practice, the bond strength varies between $5t/m^2$ and $15t/m^2$ (Table 17.4); the lower value is for stiff clays and the higher value for sound rock. It is the recommended practice to proof-test the anchors to at least 1.5 times the design load. A linear load-displacement relationship up to this load may be used as an accepted criterion. Alternatively, the anchors may be required to hold the design load without appreciable relaxation.

17.3.4 *Design of Reinforced Earth Walls-Empirical Method*

Use of the reinforcement in the backfill results in substantial increase in its overall cohesive strength (c). As such reinforced earth may be treated as soil having a very high value of cohesion. The reinforced earth slope can, therefore, stand unsupported just like hard clay. The reinforcement element is generally geogrid made of plastic mesh or plastic fabric. The manufacturing companies provide all the details of strengths of reinforcement grids and design charts. Figure. 17.14 shows a typical earth-reinforced wall that is considered suitable for low volume hill roads. The principles of design are given below.

i) The length of reinforcement (L) is generally taken to be 0.7-0.8 times the wall height and is kept at more than 4m. Figure 17.16 gives a ratio L/H for a different face or slope angle and an angle of internal friction for dry cohesionless backfill.

ii) Slope angle ß should be adopted as 70° or 3:1 for ease of construction. A too steep wall is difficult to construct without form work.

iii) Provide a drainage layer of gravels to intercept seepage in the backfill. In dry backfill or free draining backfill, it is not required.

iv) Spacing of reinforcement grids generally varies from 20 to 100 cm.

v) Approximate depth (Zi) of the reinforcement layer is obtained from Equation 17.34 (Fig.17.15). It is assumed that height of the wall is $H + 1^m$ to account for surcharge load due to traffic.

The coefficient of active earth pressure (KA) may be determined from Figure 17.16 for dry cohesionless backfill. The unit weight and ϕ of the backfill or soil are taken from Table 9.3 of Chapter 9. The safe design tensile strength of reinforcement (ft) is defined as:

$$ft = \frac{f_k}{F_k F} \qquad (17.32)$$

where,

f_k	=	characteristic tensile strength of reinforcement grid (layer) per unit length of wall,
F_k	=	1.5 (overall factor of safety of wall),
F	=	partial factor of safety for geogrid depending upon backfill,
	=	1.1-1.6 for fine-grained soil, and
	=	1.6 for gravel.

vi) Place a couple of additional grids of reinforcement in the top part of the backfill and adjust the spacing of lower grids also such that their spacing is within limit (20-100cm) and is a multiple of a specified thickness of compacted layers (e.g., 15cm)

vii) A backfill of silt or clay is not recommended for a reinforced wall. Plasticity index of fines in the backfill must be less than 6 in any case. Maximum boulder size should not be more then 125cm.

Construction of Reinforced Earth Retaining Wall

i) The first layer of reinforcement grid is laid after cleaning the foundation area of loose soil/logs/boulders, etc. (Fig. 17.16). The direction of the grid should be as assumed in the design. Then inside the backfill use U-shaped clips to maintain positions.

ii) The backfill should be compacted well in horizontal layers of 15 to 30cm. It is important to ensure that backfill is free of fines to keep it free draining. Do not use boulders longer than 125cm in size.

iii) Place boulders or gravel (thickness 25 cm) at the front face where the grid is to be wrapped up. It would be better if jute fabric was wrapped up inside the grid to prevent soil from coming out of the grid.

iv) Place the next layer of the secondary grid over the compacted backfill and tie it to the main reinforcement grid with the help of high strength plastic wires (called braid and shown in Fig. 17.14).

v) Another layer of backfill is laid and compacted. Care should be taken to ensure that grid is not damaged and displaced during compaction. For Himalayan rural roads, manual compaction is suggested.

vi) After the required thickness of backfill is laid, wrap up the first layer of the main grid as shown in Figure 17.14.

vii) Repeat the above steps to lay the next main grid and then the secondary grid.

viii) In this way the entire wall of reinforced earth is constructed.

ix) In the case of seepage problems, the drainage layer is spread over the first main layer of grid, then the backfill is laid and compacted. At the end of the grid, a drainage layer is also placed as shown in Figure 17.14.

x) Seeds are planted inside the face of the wall to grow grass on the wall face. This protects the wall from erosion, theft, rodents, and fire. Grass also restores the beauty of the terrain. It prevents local bulging of the grid.

xi) The tolerances in the position of the grids are half the thickness. Use form work of boards and logs to retain a steep wall face during construction. Good compaction of gravel/boulders at the face is required to keep a steep face stable.

Advantages

(i) It may take less time to construct the reinforced earth wall than gabions.

(ii) Excavated material from the cut may be used as backfill material provided it is not silt or clay and does not have fines. This is a major advantage where boulders or rock blocks are not available for gabions and dry stone.

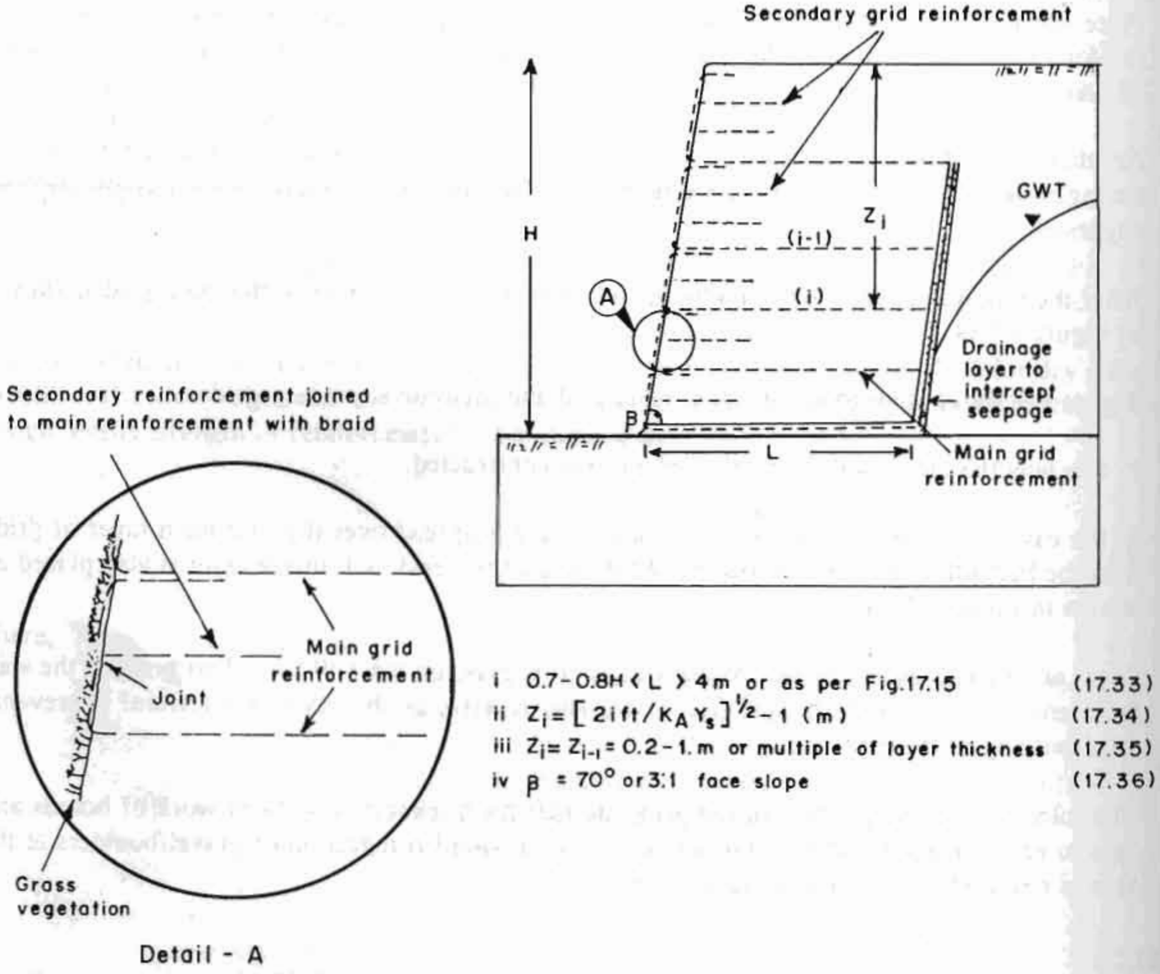


Fig. 17.14 Empirical design of reinforced earth retaining wall (9H < 15m)

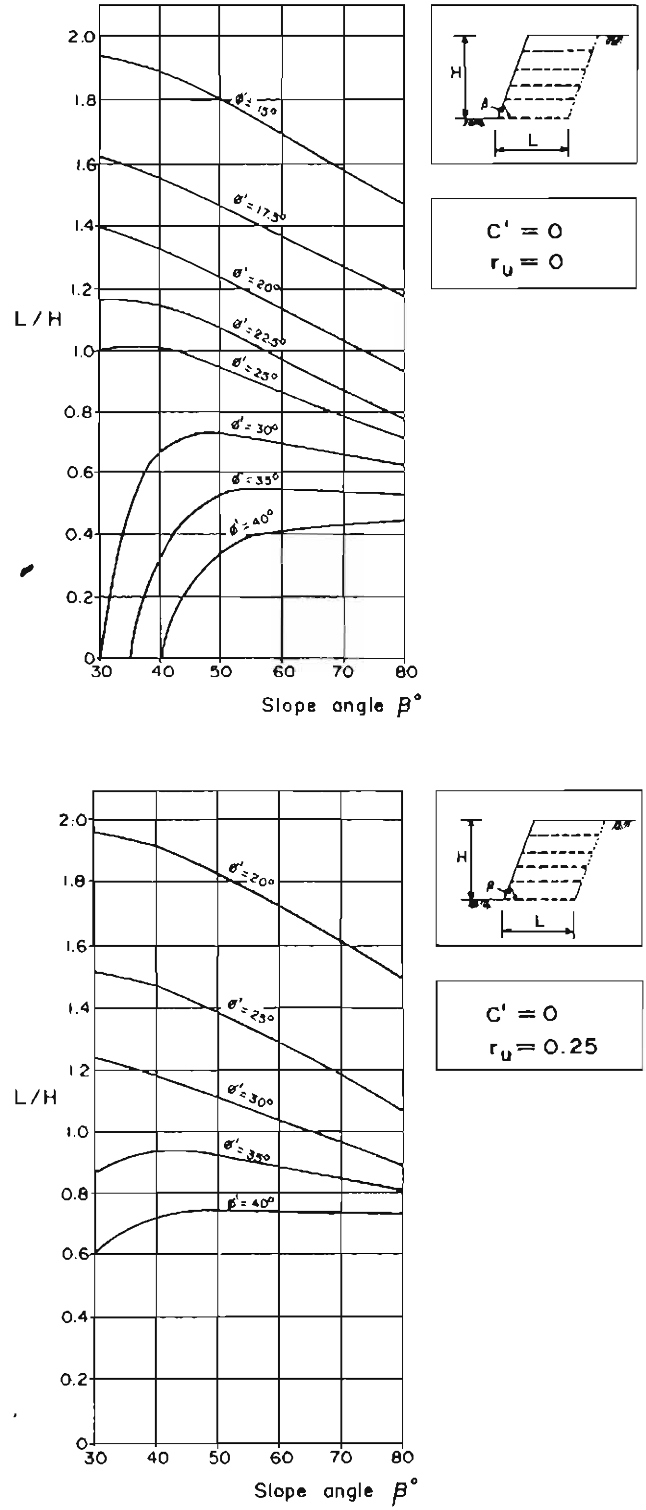
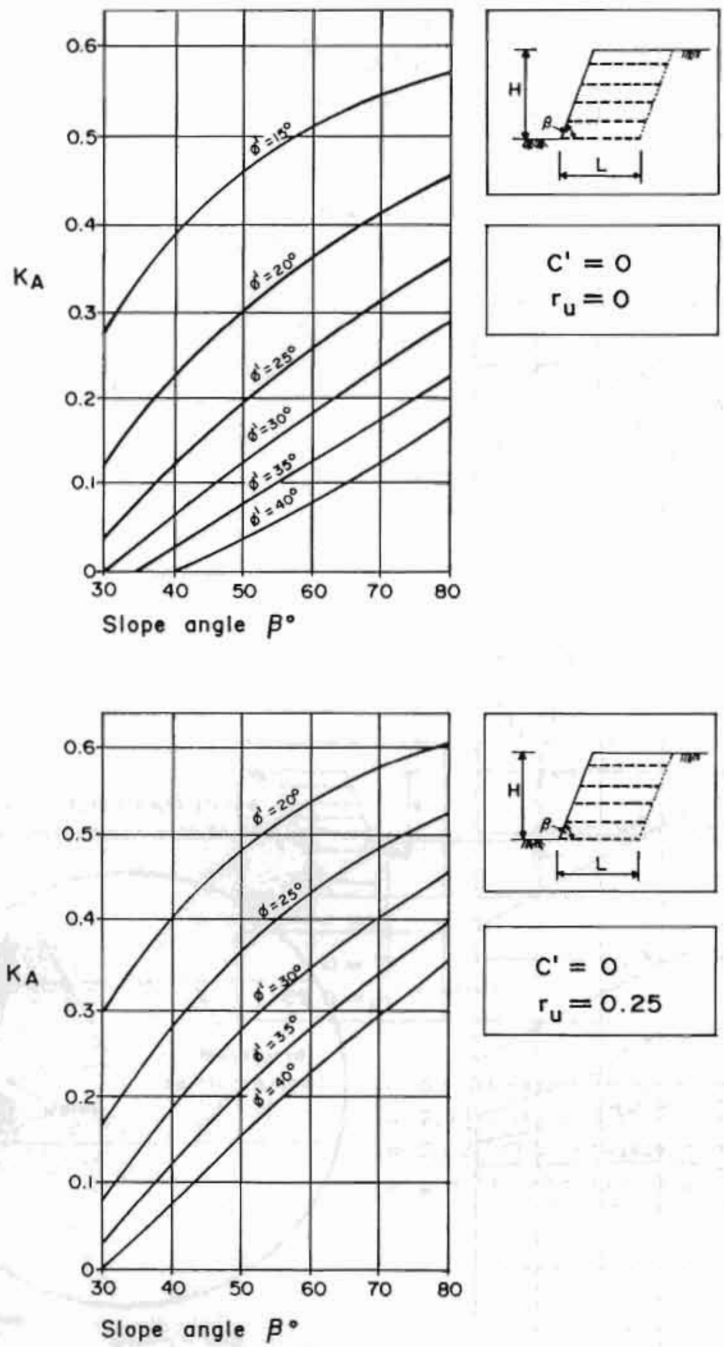


Fig. 17.15 Ratio of length of reinforcemnt L to wall height H



C' = Cohesion

r_u = Pore water pressure coefficient

$$K_A \text{ Values} = \frac{\text{Pore water pressure at depth } z}{\text{Vertical Pressure at depth } z}$$

Fig. 17.16 k_A Values