Chapter 7

STEREOGRAPHIC PROJECTION

7.1 INTRODUCTION

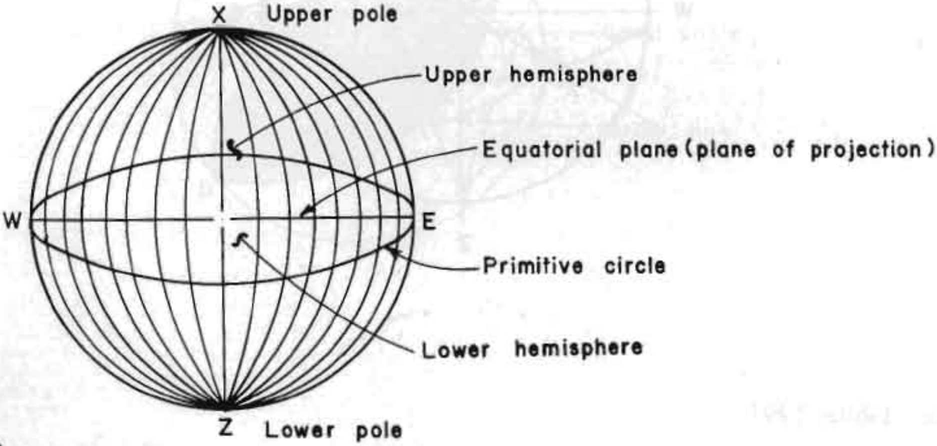
Stereographic projection is one of the convenient methods of projecting the linear and planar features. This method is used exclusively for the determination of the angular relationship among the lines as well as planes. In geotechnical engineering, it provides a quick and reliable picture of the discontinuities and their intersections. It is also used for the estimation of cut slope angle, for the preparation of hazard maps, and for the estimation of safety factors.

The stereographic projection is a projection of the sphere. The sphere is divided into two equal hemispheres by a horizontal (equatorial) plane and the upper and lower poles are fixed as shown in Figure 7.1. The plane of projection is the equatorial plane itself. The circumference of the equatorial plane is called the primitive circle. For the stereographic projection one of the hemispheres is chosen. Here the technique of projection on the upper hemisphere is discussed. The principle of projection is the same for the lower hemisphere.

7.2 PROJECTION OF A LINE

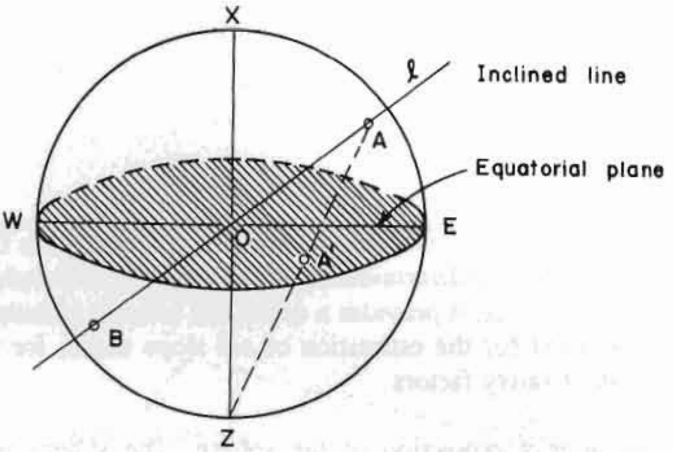
Any required line **l** is moved parallel to itself in such a way that it passes through the centre of the sphere. In doing so the line will pierce the sphere in two diametrically opposite points **A** and **B** (Fig. 7.2). For the upper hemispherical projection, the upper point is projected down to the equatorial plane as the point of intersection A' of the line joining the point A with the lower pole **Z** (Fig. 7.2).

If the line is vertical, the projection will be at the centre **O** of the sphere. If the line is horizontal, the projection will be at the primitive circle in diametrically opposite points n' and n'' (Fig. 7.3a). All the inclined lines will be projected between the primitive circle and its centre (Fig. 7.2 and 7.3b).



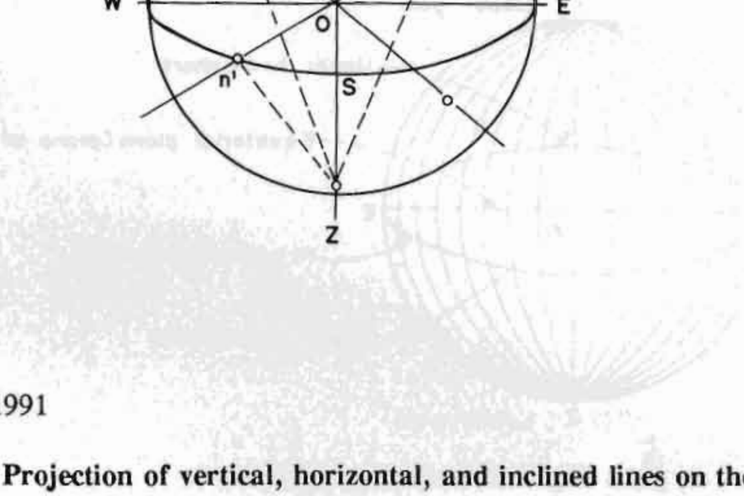
Source: Dhital 1991

Fig. 7.1 Projection sphere



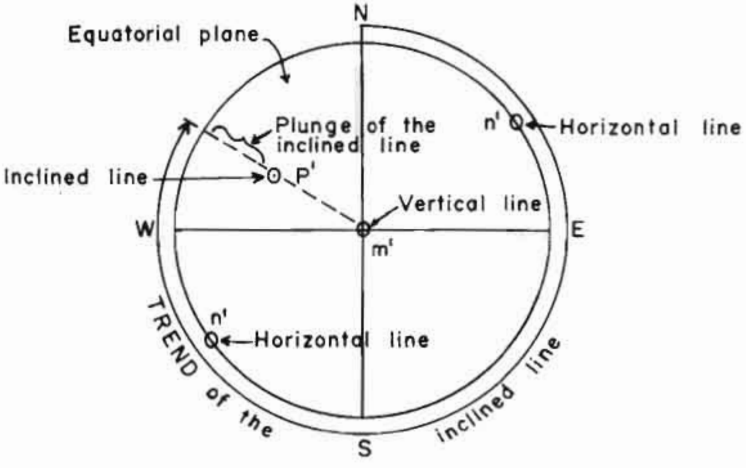
Source: Dhital 1991

Fig. 7.2 Projection of an inclined line on the upper hemisphere



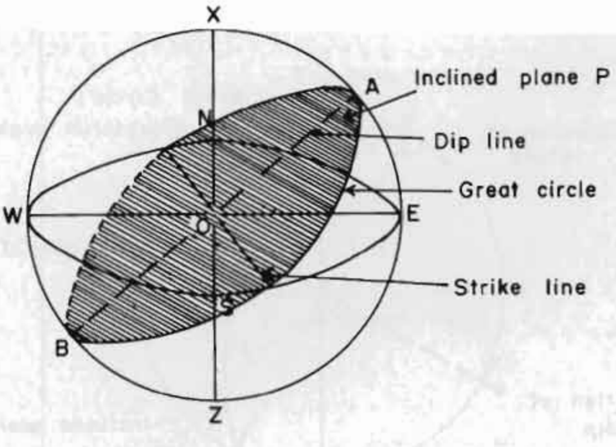
Source: Dhital 1991

Fig. 7.3(a) Projection of vertical, horizontal, and inclined lines on the upper hemisphere



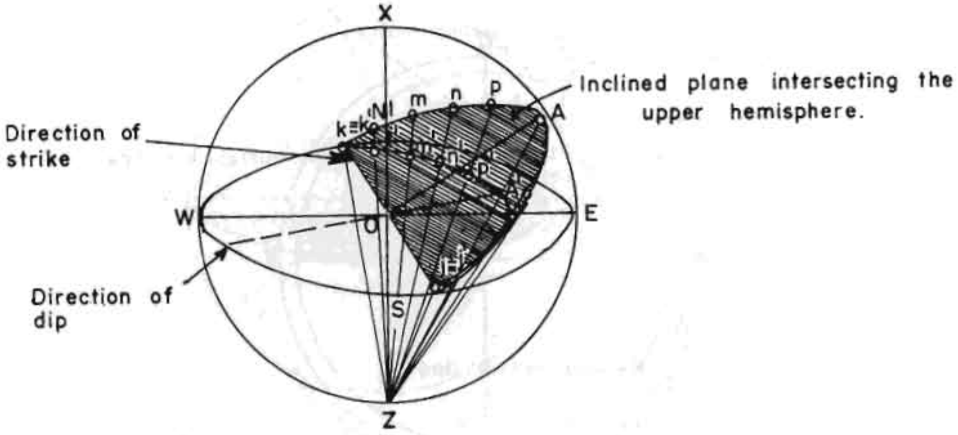
Source: Dhital 1991

Fig. 7.3(b) Stereographic projection of the horizontal, vertical, and inclined lines of Fig. 7.3(a)



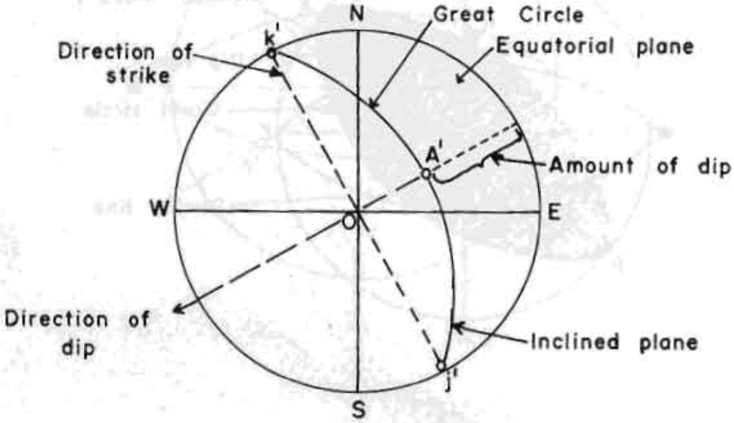
Source: Dhital 1991

Fig. 7.4 Intersection of an inclined plane



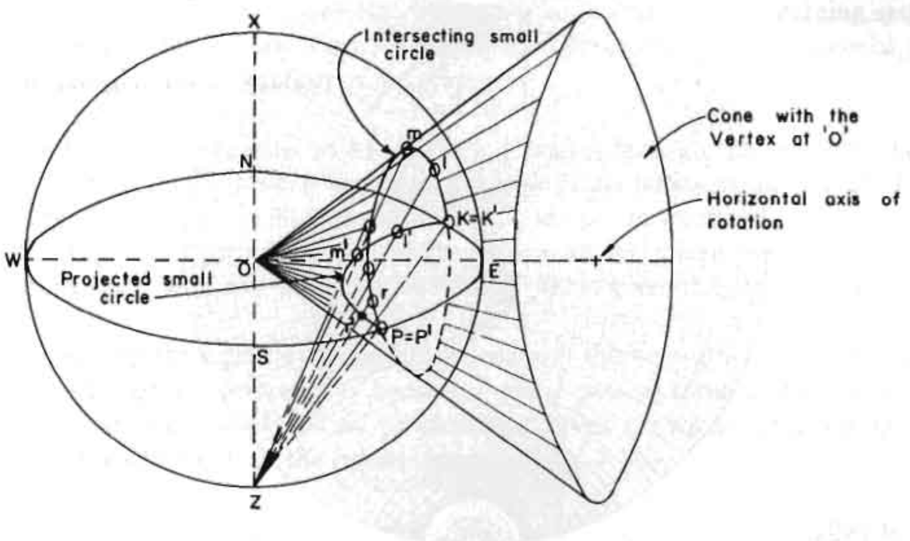
Source: Dhital 1991

Figure 7.5a Projection of an inclined plane, intersecting the upper hemisphere



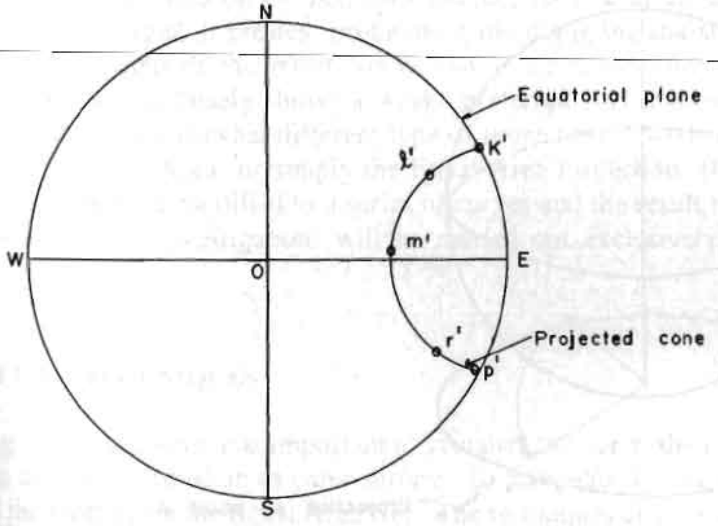
Source: Dhital 1991

Fig. 7.5(b) The stereographic projection of an inclined plane of Fig. 7.5(a)



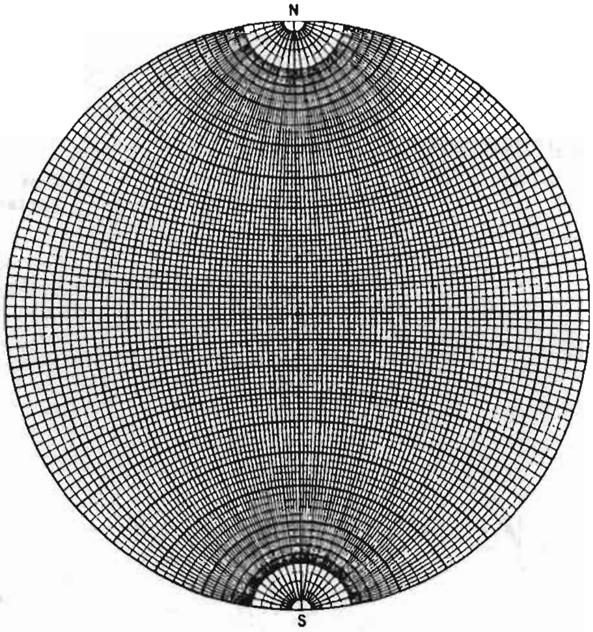
Source: Dhital 1991

Fig. 7.6(a) Projection of a cone on the upper hemisphere



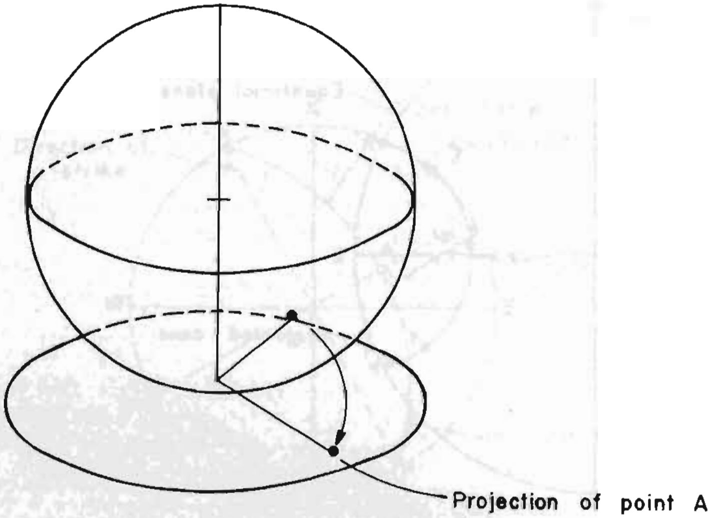
Source: Dhital 1991

Fig. 7.6(b) The stereographic projection of the cone



Source: Krähenbuhl and Wagner 1983

Fig. 7.7 Wulff Stereographic Net



Source: Rockslides: USDT 1981

Fig. 7.8 Method of projection for equal-area net

7.3 PROJECTION OF A PLANE

A plane is projected in the same way as the line. We can imagine the plane containing several lines that pass through the centre of the sphere. Each of the lines is projected on the equatorial plane and the trajectory of them will give the projection of the plane.

In general, a plane **P** is moved parallel to itself unless it passes through the centre of the sphere. The intersection of the plane with the sphere is a great circle with the radius equal to that of the sphere or primitive circle (Fig. 7.4). Then, each of the points, **k**, **l**, **m**, **n**, etc from the upper hemispherical part of the great circle (Fig. 7.5a), is joined by straight lines with the lower pole **Z** and the trace of the intersecting points, k', l', m', n', etc on the equatorial plane, gives the projection (Fig. 7.5a and b).

Any vertical plane passing through the centre will bisect both the hemispheres. Its projection will be a straight line passing through the centre. Any horizontal plane passing through the centre is projected as the primitive circle itself. Inclined planes are projected as curves known as great circles with the ends at the diametrically opposite points in the primitive circle (Fig. 7.5b).

7.4 PROJECTION OF A CONE

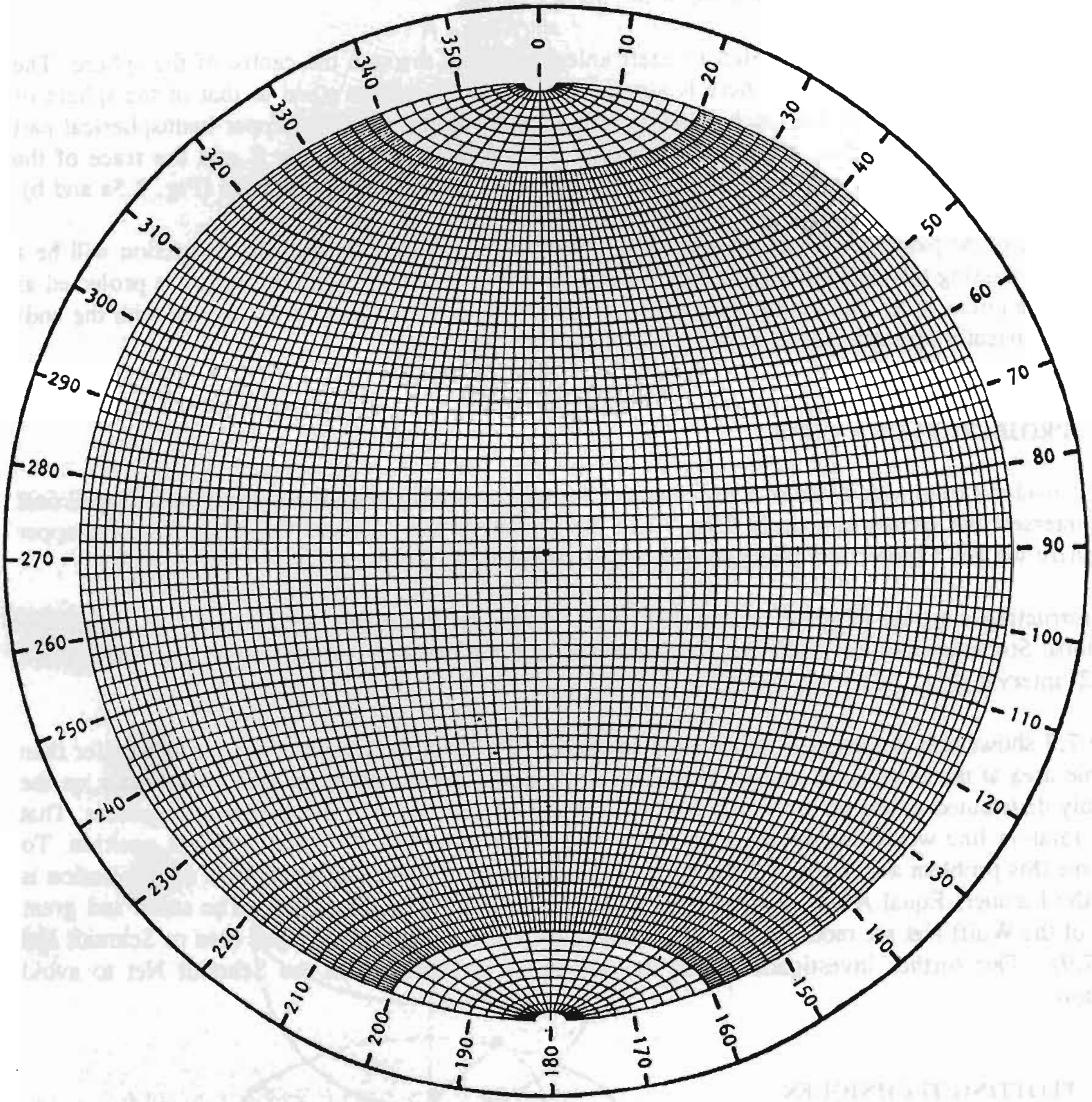
Let us consider a cone with a vertex at the centre of the sphere and horizontal axis of rotation (Fig. 7.6a). It will intersect the sphere in a circle (Fig. 7.6a). By joining all the points of the circle from the upper hemisphere we get a projection of the cone as small circles (Fig. 7.6b).

By constructing a series of great circles and small circles on the upper hemisphere, we obtain the Meridional Stereographic, or Wulff Net, or Stereonet. In it the two types of curves are generally drawn every 2° interval (Fig. 7.7).

Figure 7.7 shows that the area on the net, for example, 10°x10° in the centre of the net is smaller than the same area at the margin. It creates problems while doing the statistical analysis of the data, as the randomly distributed points on the Wulff Net will show a concentration of the points at the centre. That is, the random line would falsely show a weak preferred orientation in the vertical position. To overcome this problem a somewhat different type of projection is needed. This method of projection is called the Lambert Equal Area, or simply the Equal Area Projection. (Fig. 7.8). The small and great circles of the Wulff Net are modified to a series of curves and the result is the equal area or Schmidt Net (Fig. 7.9). Our further investigations will be carried out exclusively on the Schmidt Net to avoid confusion.

7.5 PLOTTING TECHNIQUES

When plotting on the stereonet it is important to visualize the net as the convex-upwards hemisphere and to imagine the curves inscribed on its outer surface. To make the plotting visual, several figures are given together with the plotting on the Equal Area Net. The techniques discussed here are modified after Ragan (1985).



Source: Rock Slopes: USDT 1981

Fig. 7.9 Equal Area or Schmidt Net

7.5.1 ***Plotting a Line*** (Fig. 7.10)

Given a line with trend = 40° and plunge = 46°:

1. with the tracing paper on the Equal Area Stereonet trace the primitive circle and make a small tick mark and label it **N**; this is the first step in all works on the stereonet;

2. to locate the trend of the line, count off 40° clockwise from **N** and make the second small mark on the primitive circle at this point;

3. revolve this trend mark about the centre of the net to the north point (or any other straight line such as East-West and North-South) of the net;

4. count off 46° from the primitive, inwards along the diametrically opposite (Southern) end of the North-South straight line and mark the point **P**; and

5. restore the overlay to the starting position and recheck by visualization.

   Visualization: Hold a pencil over the centre of the stereonet in the direction of the given trend direction at the angle of plunge. Visualize its intersection with the upper hemisphere in the southwest quadrant.

7.5.2 ***Plotting a Plane*** (Fig 7.11)

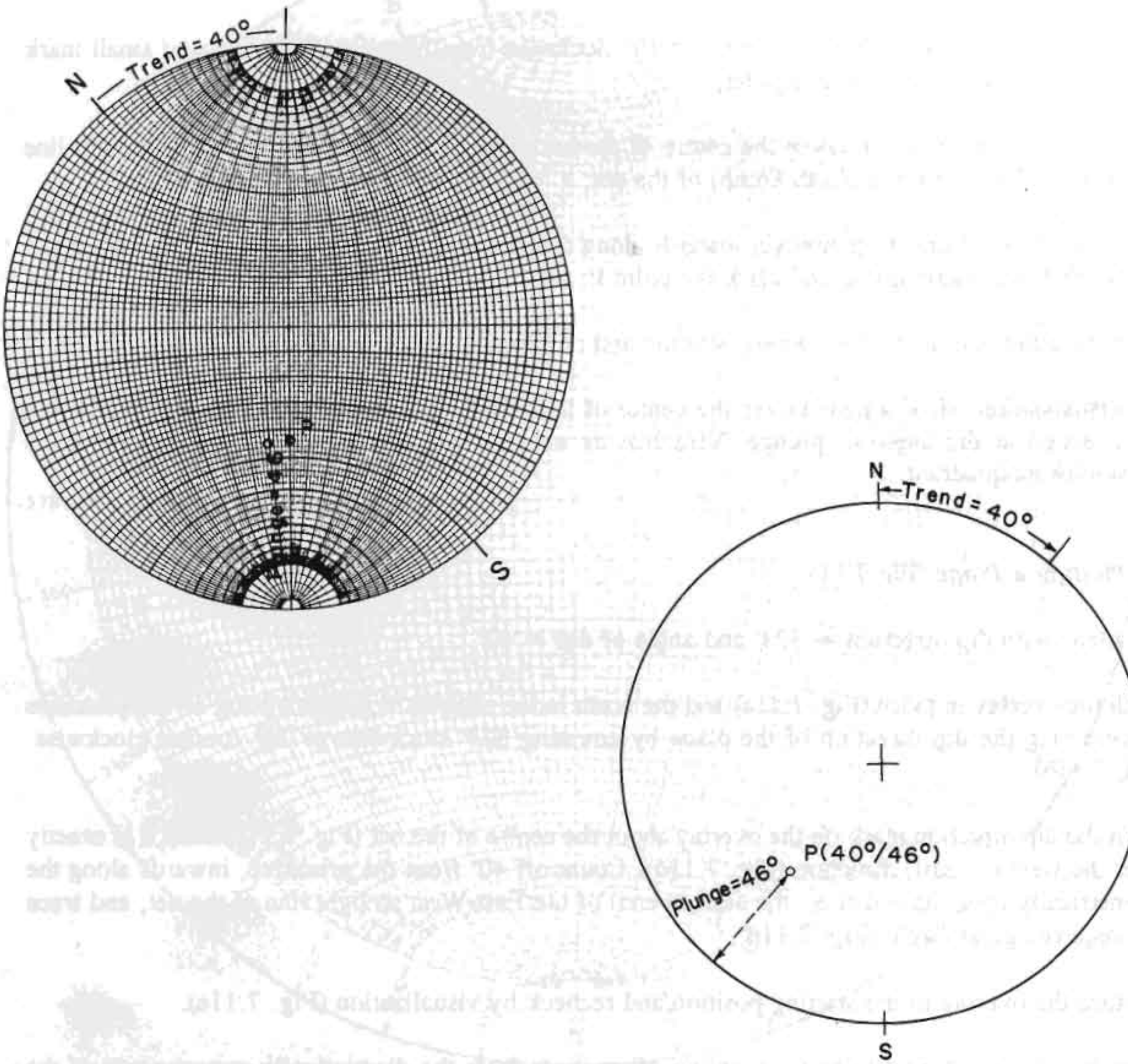
Given a plane with dip direction = 324° and angle of dip = 40°

1. With the overlay in place (Fig. 7.11a) and the north index marked **N**, locate a point on the primitive representing the dip direction of the plane by counting 324° clockwise or 36° counter clockwise. (Fig.7.11b).

2. Turn the dip direction mark on the overlay about the centre of the net (Fig. 7.11c) until it is exactly over the west (or east) direction (Fig. 7.11d). Count off 40° from the primitive, inwards along the diametrically opposite end (i.e., the eastern end) of the East-West straight line of the net, and trace the required great circle (Fig. 7.11d).

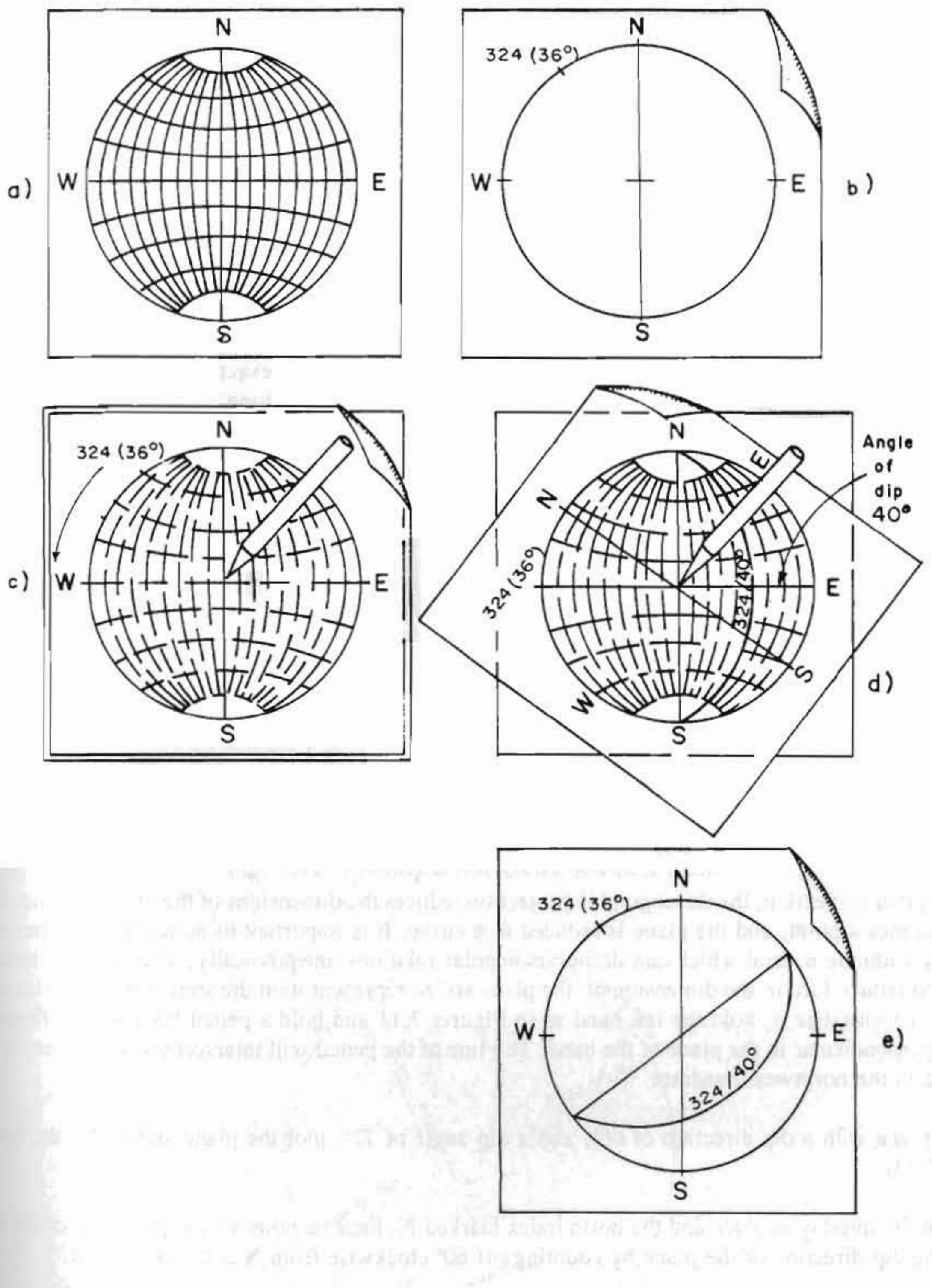
3. Restore the overlay to the starting position and recheck by visualization (Fig. 7.11e).

   Remarks: Do not forget to begin counting off inwards from the diametrically opposite end of the marked position. If you count off directly inwards from the marked position, it will be the projection on the lower hemisphere. Visualization: Hold the right hand palm upwards, over the centre of the stereonet with the fingers pointing towards 324° + 90° = 414° = 54° NE and the plane of the hand inclined 40° to the northwest (324°). The plane of the hand in this position can be imagined to extend into the upper hemisphere and intersect its surface. Its trace cuts the southeast quadrant and this is where the final plot must be.



Source: Dhital 1991

Fig. 7.10 Plotting a line



Source: Krähenbuhl and Wagner 1983

Fig. 7.11 Plotting a plane

7.5.3 *Plotting a Line Contained in a Plane*

Given a plane (sandstone bed) with a dip direction of 130° and a dip amount 55° and a line (intersection of the vertical cut slope with the sandstone bed) with a trend of 78° and lying in that plane. Find the plunge of the line (apparent dip of the bed along that cut slope).

Note: The apparent dip is the angle between the line of intersection of the sandstone bed with the vertical cut slope and any horizontal line parallel to the slope.

1. With the overlay in place (Fig. 7.12), and the north index marked N, locate a point on the primitive representing the dip direction of the plane by counting 130° clockwise from N.

2. Turn the dip direction mark about the centre of the net until it is exactly over the east (or west) direction (Fig. 7.12). Count off 55° from the primitive, inwards along the diametrically opposite end (i.e., the western end) of the East-West straight line, and trace the required great circle.

3. Restore the overlay to its starting position and count off 78°' clockwise from N and make a tick mark.

4. Rotate the overlay about the centre and coincide the tick mark with the nearby diameter and locate the required point (line) in the previously traced great circle. In the same position count off the plunge of the line which is equal to the angle between the tick-marked point in the primitive and point (line) located in the great circle.

5. Restore the overlay to the starting position and recheck by visualization (Fig. 7.12).
   Result: the plunge is 41°

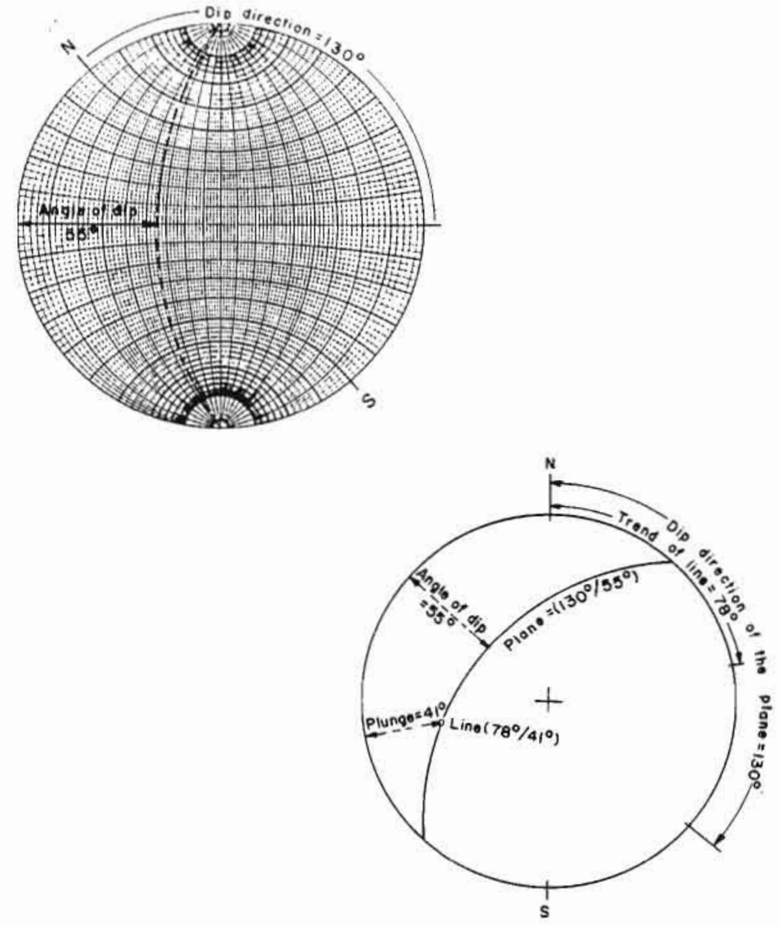
7.5.4 *Plotting a Pole*

Like any other projection, the stereographic projection reduces the dimensions of the object by one. Here, a line becomes a point, and the plane is reduced to a curve. It is important to notice the fact that every plane has a unique normal which can define its angular relations unequivocally. Therefore, it becomes possible to reduce further the dimension of the plane and to represent it on the stereonet by a point called the pole. To visualize it, hold the left hand as in Figure. 7.11 and hold a pencil between the fingers so that it is perpendicular to the plane of the hand. The line of the pencil will intersect the upper hemisphere at a point in the northwest quadrant.

Given a plane with a dip direction of 60°, and a dip angle of 72°, plot the plane and locate the pole to it (Fig. 7.13).

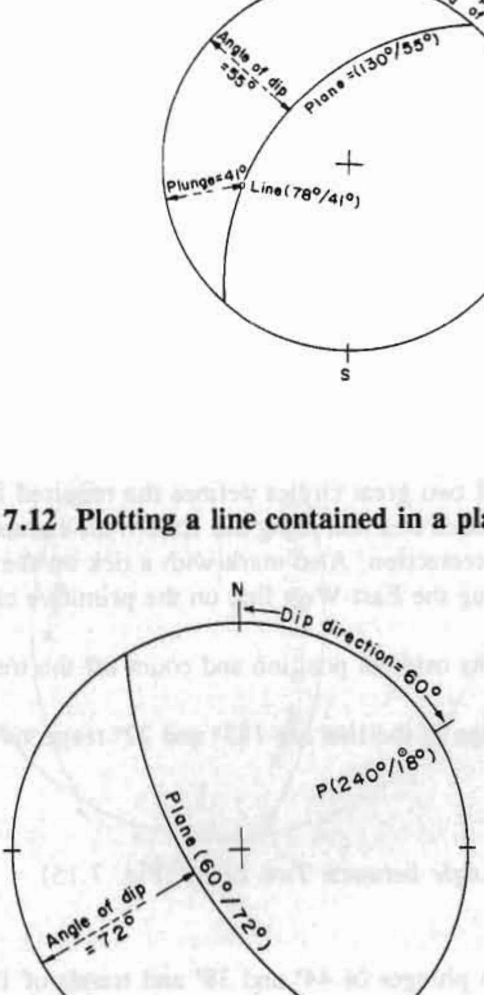
1. With the overlay in place and the north index marked **N**, locate a point on the primitive circle equal to the dip direction of the plane by counting off 60° clockwise from **N** and tick mark it.

2. Rotate the overlay about the centre until the tick mark is over the East-West diameter of the net and, by counting off 72° inwards from the opposite end of the diameter, trace the required great circle.



Source: Dhital 1991

Fig. 7.12 Plotting a line contained in a plane



Source: Dhital 1991

Fig. 7.13 Plotting the pole to a plane

3. As the pole is everywhere located 90° from the plane, count off 90° towards the tick-marked point from the great circle and locate the pole (Fig. 7.12).

4. Restore the overlay to its original position and visualize the plotting by holding the left hand over the centre of the net and tilting it towards **NE** (60°) by 72°, while the fingers will point towards 60°-90° = -30° = 330° **NW** (the strike). In this position, the plane will be plotted in the **SW** quadrant and the pole in the **NE** quadrant.

Note that it is easier to plot the pole by rotating the overlay so that the dip direction mark coincides with any nearby diameter and by counting off the dip amount (here, 70°) from the centre of the net towards the tick mark (dip direction). Check the previous results with this method.

Result: The position of the pole is 240°/18°.

7.5.5 *Plotting the Line of Intersection of Two Planes* (Fig 7.14)

Given two intersecting planes (joints), having dip amounts of 40° and 60° and a dip direction of 120° and 260° respectively, it is required to find the plunge and the trend of the line of intersection (wedge axis).

1. With tracing paper over the stereonet trace the primitive circle and mark the north point. Measure off the dip direction of 120° clockwise from **N** and mark this position on the primitive.

2. Rotate the overlay about the centre of the net until the dip direction mark lies on the East-West diameter of the net. Measure 40° from the primitive and trace the required great circle.

3. Rotate back the tracing paper to its original position and repeat the same for the second plane.

4. The point of intersection of two great circles defines the required line. Rotate the overlay until the intersection of the two great circles lies along the East-West diameter of the stereonet and measure the plunge of the line of intersection. Also mark with a tick on the overlay at the diametrically opposite direction (trend) along the East-West line on the primitive circle.

5. Rotate the tracing back to its original position and count off the trend of the line.

   Result: The trend and plunge of the line are 183° and 22° respectively.

7.5.6 *Determination of the Angle between Two Lines* (Fig. 7.15)

Given: Two lines in space with plunges of 44° and 38° and trends of 120° and 220° respectively. Find the angle between these lines:

1. mark the points A and B on the overlay as described under 7.5.1;

2. now rotate the overlay until these two points lie on the same great circle of the stereonet; and

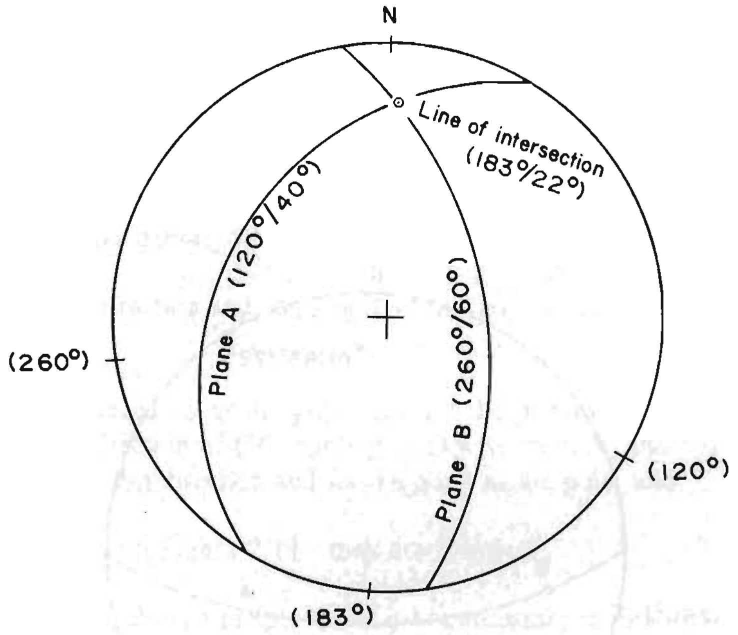
3. determine the angle between the two lines by counting the small circle divisions between A and B along the great circle.

Note that the great circle on which A and B lie defines the plane that contains these two lines.

Result: The angle is found to be 72° and the dip direction and dip amount of the plane containing the line are 165° and 54° respectively.

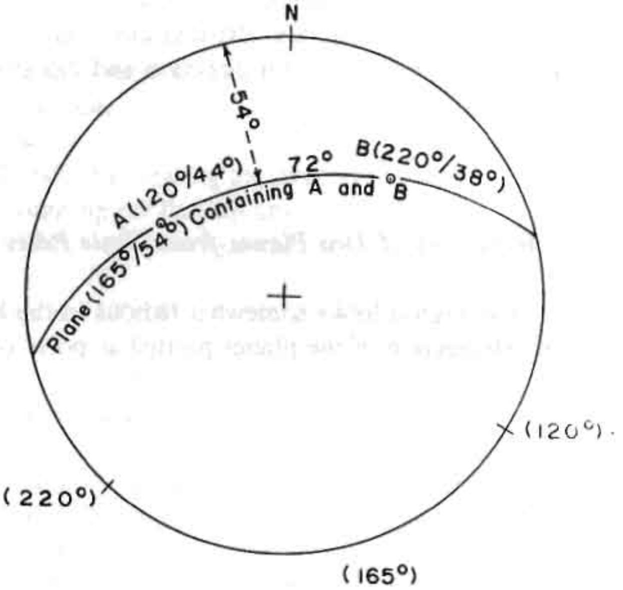
7.5.7 *Plotting the Line of Intersection of Two Planes from Their Poles* (Fig. 7.16)

This is an alternative to 7.4.4. Although it looks somewhat tedious in the beginning, it is a very useful method for plotting the lines of intersection of the planes plotted as poles obtained from the field data.



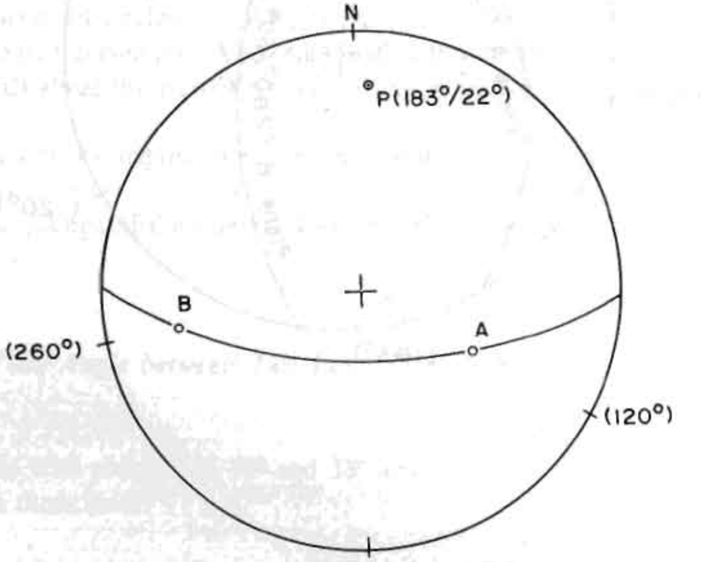
Source: Dhital 1991

Fig. 7.14 Plotting the line of intersection of two planes



Source: Dhital 1991

Fig. 7.15 Determination of the angle between two lines



Source: Dhital 1991

Fig. 7.16 Plotting the line of intersection of two planes from their poles

Given: the same two planes of 7.5.5 with dips of 40° and 60° and dip directions of 120° and 260° respectively. Plot their poles and find their line of intersection:

1. plot the poles to the planes as discussed in 7.5.4 and mark them as **A** and **B**;

2. rotate the overlay until both poles lie on the same great circle. This great circle defines the plane that contains the two normals to the planes; and

3. find the pole of this plane by measuring the dip on the East-West diameter of the net, locate it as described in 7.5.4., and mark it as **P**.

Note that this pole **P** is a pole to a plane which passes through the poles of two other planes and, therefore, is a common normal to both poles **A** and **B**. But, as the pole **P** is normal to **A** and **B**, it must be parallel to the given planes and hence to their line of intersection.

Result: The trend and plunge of the line of intersection are 183° and 22° respectively.

7.6 POLE NET

A pole net is used for plotting a large number of poles to the planes measured from the field. To plot several planes on the same diagram is very difficult. The diagram looks very cumbersome and there is no way of statistical analysis. On the other hand, the poles provide a quick and convenient way of plotting the field data and carrying out their statistical analysis.

A polar stereonet or pole net (Fig. 7.17) is obtained by projecting, on the equatorial plane the vertical planes of varying strike and the small circles of varying radius centred in the upper pole.

7.6.1 *Plotting the Pole on a Pole Net* (Fig 7.18)

Given a plane with a dip direction and dip angle of 60° and 72° respectively, plot its pole on the pole net:

1. with tracing paper placed over the pole net trace the primitive circle and mark the N direction, measure off the dip direction of 60° counting clockwise from **N**, and count off 72° from the centre of the net towards the dip direction and fix the point as the pole; and

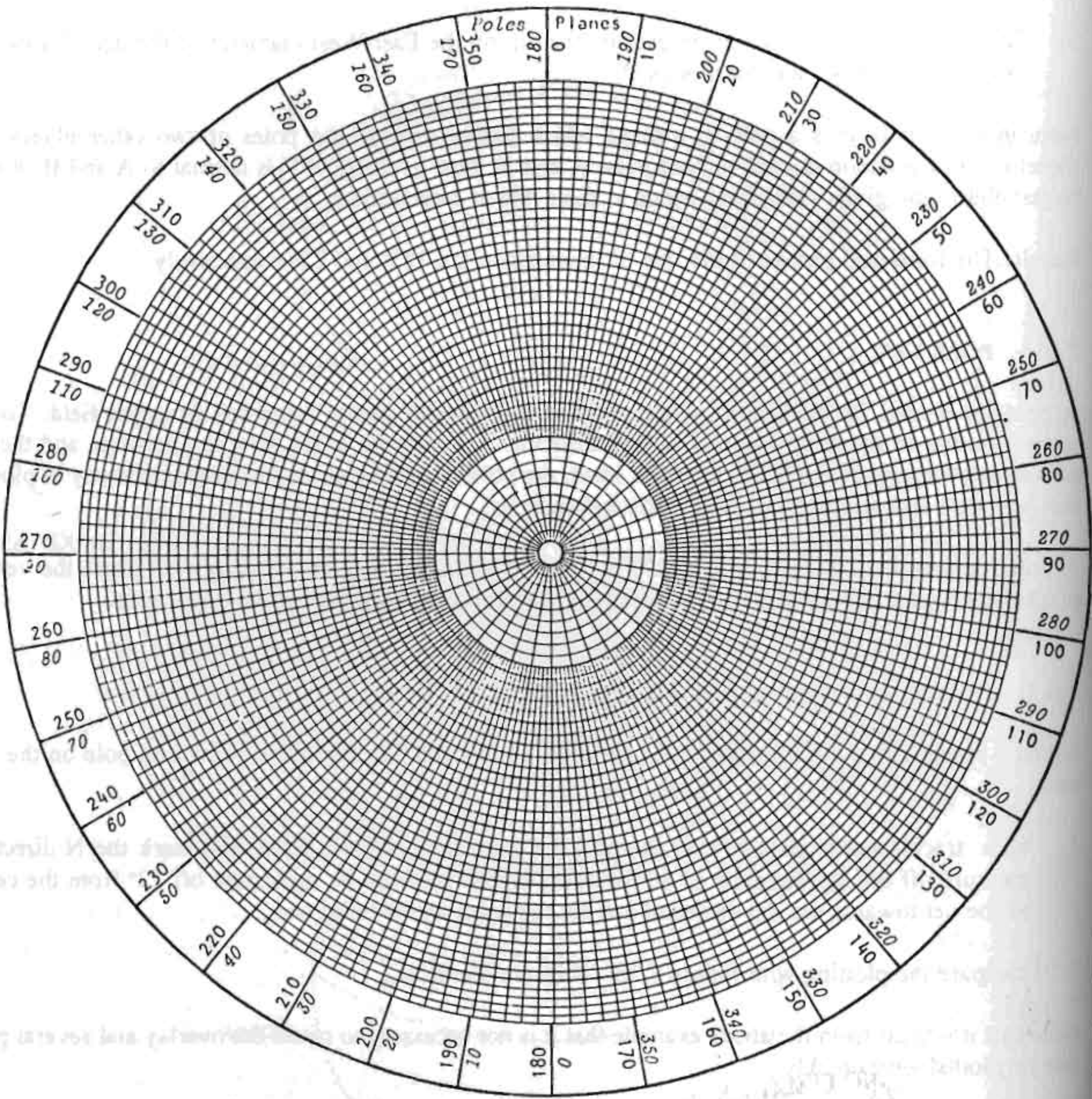
2. compare the plotting with Figure 7.13 - they are identical.

Note that it is clear from the above example that it is not necessary to rotate the overlay and several poles can be plotted very quickly.

7.7 CONTOURING FIELD DATA AND STATISTICAL ANALYSIS

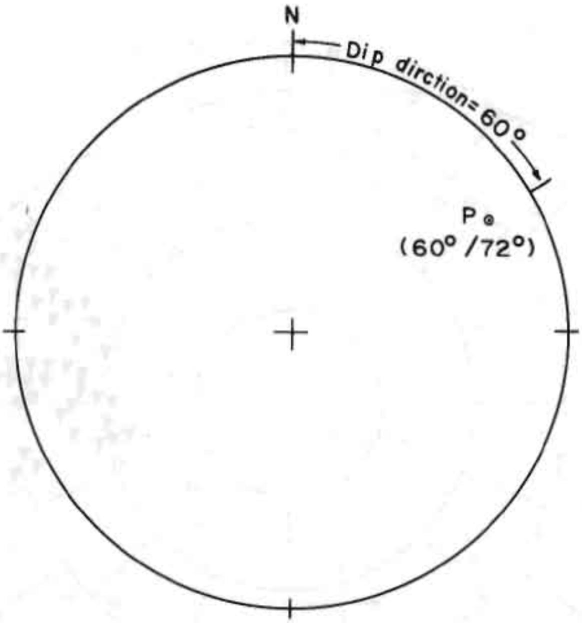
Occasionally it is necessary to plot the discontinuity pattern accurately. For that purpose, pole concentrations need to be contoured in order to obtain the statistical mean for each set of the discontinuity.

A plot of 351 poles of bedding planes and joints and of one fault (United States Department of Transport [USDT] 1981) in a hard rock mass is given in Figure 7.19.



Source: USDT 1981

Fig. 7.17 The pole net



Source: Dhital 1991

Fig. 7.18 Plotting the pole on a pole net

The following procedure is recommended for contouring the poles.

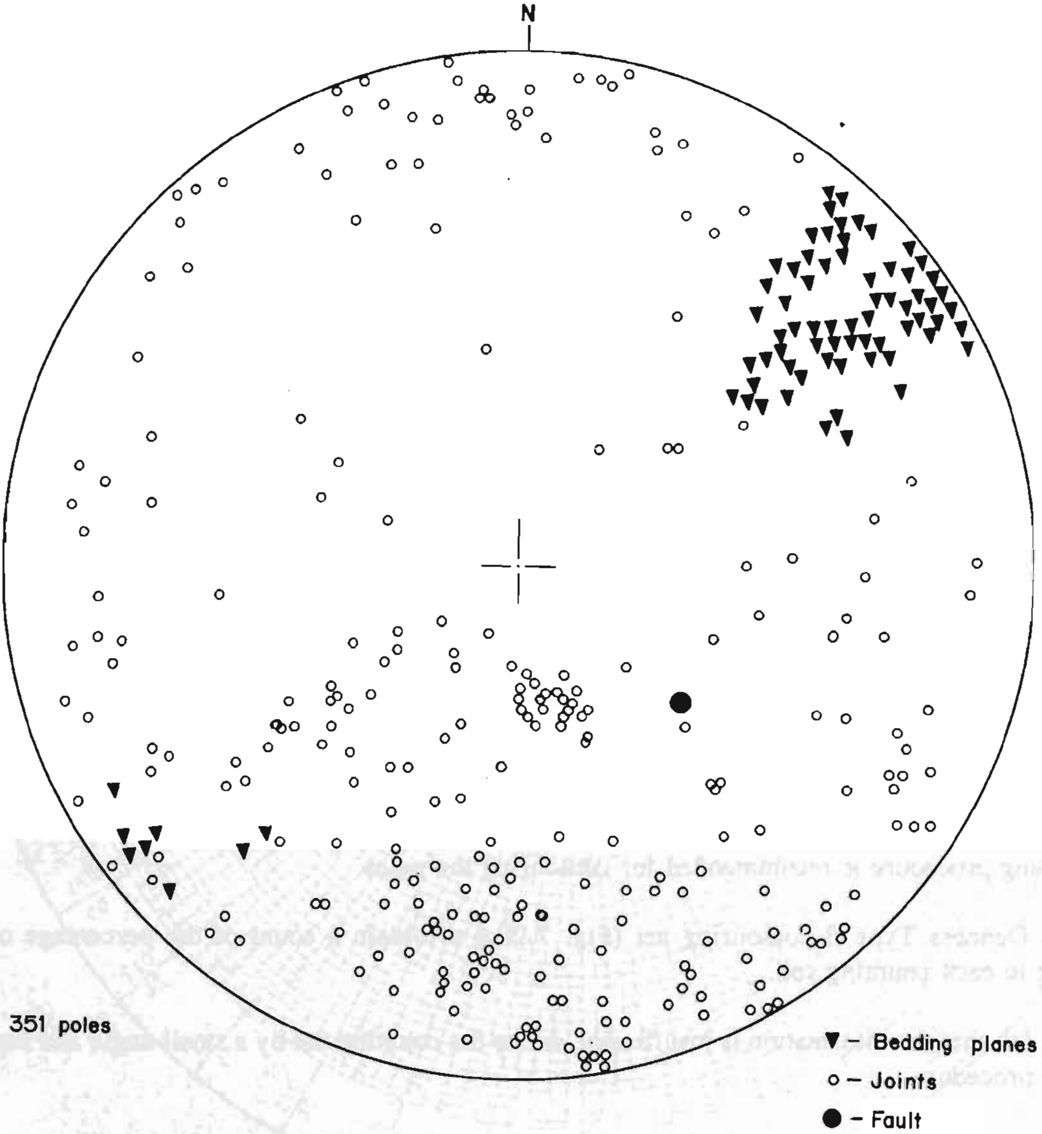
1. Use a Denness Type B contouring net (Fig. 7.20a) to obtain a count of the percentage of poles falling in each counting cell.

2. If it is felt that the information is insufficient, rotate the counting net by a small angle and repeat the above procedure.

3. Draw very rough contours on the basis of the pole counts noted on the tracing paper.

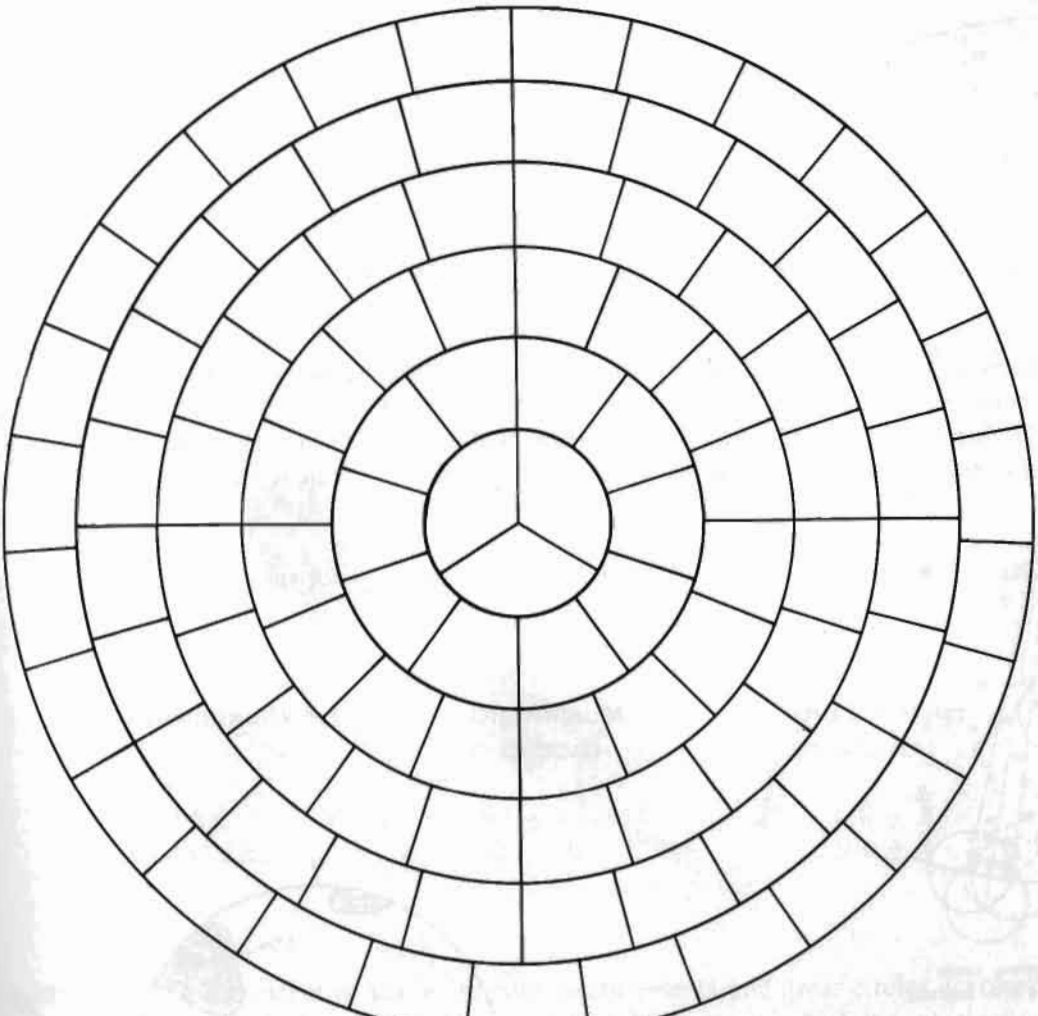
4. Use the circle counter (Fig. 7.20b) to refine the contours starting with low value contours and working inwards towards the maximum pole concentration.

Note: In order to construct a circle counter, trace the pattern given in Figure 7.20b on to a transparent plastic sheet. Make two little holes at the centre of each circle and draw a straight line through the centres of the two circles.



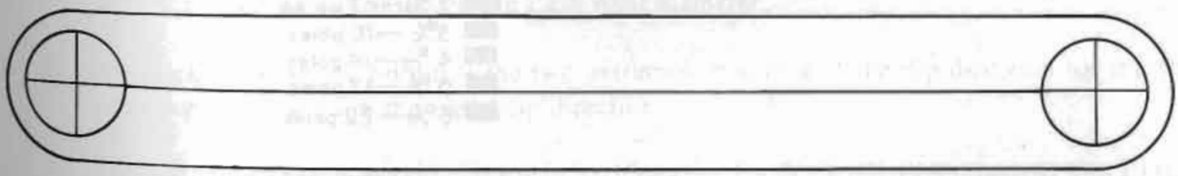
Source: USDT 1981

Fig. 7.19 Plot of poles of discontinuities



Source: USDT 1981

Fig. 7.20(a) Denness Type B curvilinear cell mounting net

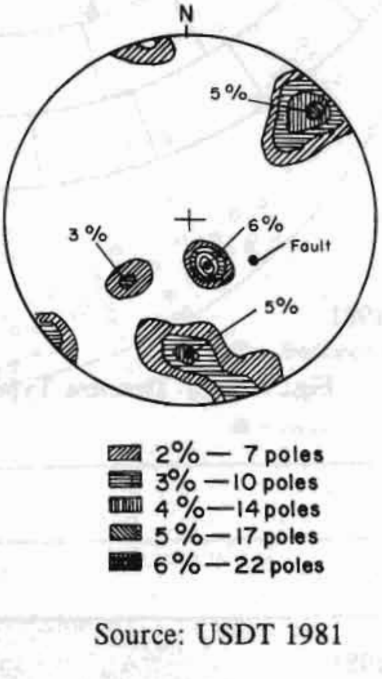


Source: USDT 1981

Fig. 7.20(b) Counting circles for use in contouring pole plots

Source: USDT 1981

Fig. 7.21 Contouring by counting circles



Source: USDT 1981

Fig. 7.22 Contour diagram obtained from contouring of poles in Figure 7.19

Figure 7.21 illustrates the use of the circle counter for the construction of 3 per cent or 10 to 11 poles. Move one of the small circles around the poles, until 10 to 11 poles are enclosed by it, and mark the centre of the circle by a pencil. Continue this process till a complete contour is drawn by the locus of the points. If the circle lies partly out of the primitive circle, the total number of poles falling in the circle is added to the number of poles reappearing at the diametrically opposite end (Fig. 7.21).

Figure 7.22 is the contour diagram obtained from the contouring of 351 poles given in Figure 7.19.

7.8 DETERMINATION OF EXTENT OF SCATTER AROUND THE MEAN POLE OR GREAT CIRCLE POSITION

When the mapping covers a large area the scatter in the dip and dip direction measurements is considerable and must be taken into account in the analysis. After contouring the diagram as explained in Figure 7.7, a mean pole position for more than one set of discontinuities is determined. The procedure for the determination of the scatter of the line of intersection of two sets of discontinuities is obtained as follows:

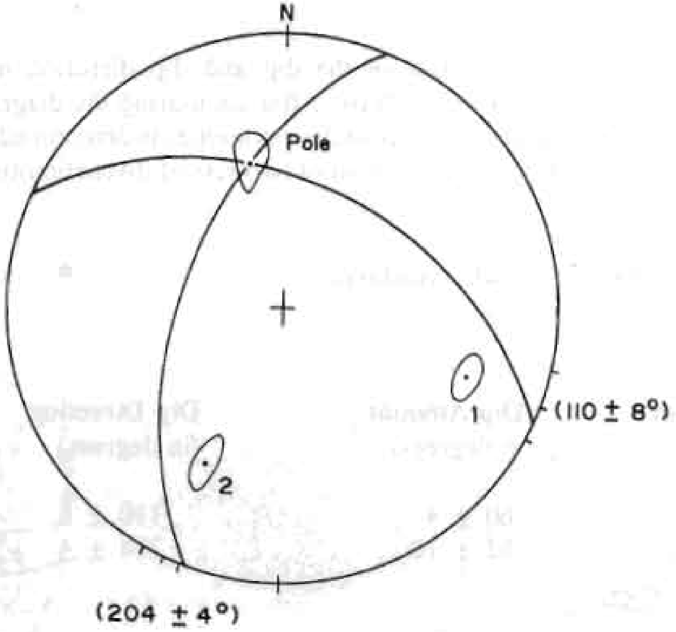
Given the mean pole positions for two sets of discontinuity:

Discontinuity Set	Dip Amount (in degrees)	Dip Direction (in degrees)
First Set	60 ± 4	110 ± 8
Second Set	52 ± 10	204 ± 4

It is necessary to plot the extent of scatter in pole measurements and great circles corresponding to the most probable pole positions. It is also necessary to find the extent to which the intersection point (line) is influenced by the scatter around the pole points (Fig. 7.23).

1. Plot the mean pole positions and corresponding great circles of the two sets of discontinuity according to 7.5.4.
2. Tick mark the two extremes of scatter in the dip angle of dip for the first set by counting off ± 4° from the mean pole position on the East-West diameter.
3. Also tick mark on the primitive the two extremes of scatter in the dip direction for the first set by counting off ± 8° from the mean dip direction.
4. Determine the corresponding mean pole positions for the above dip directions and join all the scatters with a smooth curve.
5. Repeat 2,3, and 4 for the second set and determine the scatter.

6. To obtain the influence of scatter of poles around the intersection point, rotate the overlay to coincide with a great circle containing simultaneously any outer point of the great circle. The locus of the poles, thus marked around the intersection point, defines the required influence area (Fig. 7.23).



Source: Dhital 1991

Fig. 7.23 Determination of extent of scatter in pole measurements