4. The Translog Cost Function

A production function defines the physical relationship between output(s) and input(s). Consider Q as output and capital (K), labour (L), energy (E), and material (M) as aggregate inputs entering the production process then:

$$Q = f(K, L, E, M). \quad (1)$$

It is likely that producers allocate their outlays first to major groups of inputs such as labour, material, and energy. Once this decision is made and each major input budget has been allocated, the decision to purchase individual inputs within each broad group of inputs may be the second step. For example, once the energy budget has been established, producers may then decide on the amounts of wood, coal, diesel, or electricity to be purchased. In the jargon of economics, such a process implies that inputs are weakly separable. Separability of inputs means that inputs in the production function can be partitioned into subgroups so that some form of independence exists among inputs in each subgroup. As a result, inputs purchased can be written as a function of group expenditure and prices. Weak separability in inputs exists if input use in one subgroup is related to input use in another group in a fixed manner. Stated differently in terms of economic theory, weak separability implies that the marginal rate of technical substitution between two inputs in a subgroup is weakly independent of input use in another subgroup. Other forms of separability can also exist among subgroup of inputs. Global separability implies no dependence among inputs in a subgroup (case of Cobb-Douglas function). Strong separability implies that only some inputs in each subgroup will exhibit relationship among inputs in other subgroups. It is only when weak separability among a subgroups of inputs exists does subgroups aggregates exist. Since our interest in this exercise is to understand the relationship between different energy types used in the manufacturing sector, we assume that energy input is weakly separable from other inputs in the Nepalese manufacturing sector.

Since labour and material inputs are the other two inputs assumed to be used in the manufacturing sector, at least in the case of energy input, it is assumed that energy is weakly separable from materials and labour. Therefore given an aggregate production function (Eq. 1) and assuming energy to be weakly separable from labour and material aggregates, it is possible to write the production function as in Equation 2.

$$Q = f\{E(E_w, E_d, E_c, E_k, E_e), L, K, M)\} \quad (2)$$

Table 3: Energy Consumption Pattern by Type of Energy and Sector ('000 TCE)

	1981	1982	1983	1984	1985	1986	1987	1988	1989	Growth
Domestic										
Traditional Fuel:										
Wood	3110	3193	3196	3365	3455	4220	4319	4419	4500	4.73
Residue	55	56	57	384	394	620	631	643	652	36.22
Dung	21	22	21	7	69	469	480	490	498	48.55
Petroleum: LPG	0.26	0.4	0.52	0.55	2	1	1.3	1.3	2	29.05
Kerosene	31	28	28	38	40	49	53	56	52	6.68
Total Traditional	3186	3271	3274	3816	3918	5309	5430	5552	5650	7.42
Total Petroleum	31.26	28.4	28.52	38.55	42	50	54.3	57.3	54	7.07
Total Commercial	38.26	36.4	38.52	47.55	53	64	70.3	73.3	70	7.84
Total	3255.5	3335.8	3341	3902.1	4013	5423	5554.6	5682.6	5774	7.43
Industrial Sector										
Fuelwood	22	45	45	47	48	72	44	46	48	10.24
Residue						6	6	6		
Diesel	3	3	2	2	3	10	4	4	4	3.66
Fueloil	2	2	3	3	5	7	7	5	4	9.05
Kerosene						1	1	1		
Coal	22	24	24	56	46	30	33	35	30	3.95
Electricity	4	5	7	7	8	12	14	14	15	17.97
Total Traditional	22	45	45	47	48	78	50	52	54	11.88
Total Petroleum	5	5	5	5	8	17	12	10	9	7.62
Total Commercial	31	34	36	68	62	59	59	59	54	7.18
Total	53	79	81	115	110	137	109	111	108	9.31
Commercial Sector										
Fuelwood	9	19	19	20	21	13	12	13	15	6.59
Kerosene	2	1	2	2	2	3	6	13	16	29.68
LPG					1	1	1	1	1	
Diesel	3	3	2	2	3	1	2	2		-4.94
Fueloil	2	2	3	3	5	7	7	5	4	9.05
Total Petroleum	7	6	7	7	11	11	15	21	23	16.03
Coal	1	1	3	1	1	17	17	17		
Electricity	2	2	2	4	4	2	2	2	3	5.20
Total Commercial	9	9	10	14	16	1434	40	43		21.59
Total	18	28	29	34	37	27	46	53	58	15.75
Transport sector										
HSDO	47	43	49	55	58	.45	70	88	72	5.48
Motor Spirit	9	11	12	13	14	16	16	18	16	7.46
ATF	14	16	16	19	18	18	20	23	18	3.19
Total Petroleum	70	70	77	87	90	79	106	129	106	5.32
Electricity										
Coal	1	1	1	1	1	1	1	1	1	0.00
Total Commercial	71	71	78	88	91	80	107	130	107	5.26
Agriculture										
Petroleum	5	5	5	6	7	14	7	8	7	4.30
Electricity	1	1	2	2	1	2	3	6		
Total	5	6	6	8	9	15	9	11	13	12.69

Source: Water and Energy Commission 1989

Notes: LPG, HSDO, and ATF are liquified petroleum gas, high speed diesel oil, and aircraft turbine fuel respectively.

What Equation 2 says is that the production function is characterised by an aggregator energy function $E(E_w,E_d,E_c,E_k,E_e)$ and aggregate labour (L), capital (K), and materials (M) respectively. The aggregator energy function E(.) is assumed to consist of wood (w), diesel (d), coal (c), kerosene (k), and electricity (e). It is also possible to define each input in terms of its sub-inputs. For example, the labour input can be assumed to consist of skilled, semi-skilled, and unskilled labour and a similar aggregator function for labour can be defined. Since our interest in the present exercise is only to model the energy demand originating in the manufacturing sector, labour, capital, and material inputs are treated as single aggregate inputs.

Energy Sub-model

Presuming a weak separability of energy inputs in the Nepalese manufacturing sector, the energy demand can be modelled in two stages. If it is assumed that factor prices and output levels are exogenously determined, then the production function (Eq. 1) implies that duality between production and cost exists and, hence, Equation 1 can be written in terms of a cost function as in Equation 3.

$$C = g(P_K, P_L, P^E, P_M, Q). \tag{3}$$

The cost function (Equation 3) represents the minimum cost required to produce the output level Q given labour, capital, energy, and material prices (P_K, P_L, P_E, P_M) respectively. In this study the cost function corresponding to Equation 3 is represented by the translog second-order approximation or simply the translog cost function. The translog cost single output, multi-input cost function developed by Christensen, Jorgenson, and Lau (1973) can be written as:

$$lnC = \alpha_0 + \Sigma\alpha_i*(lnP_i) + \alpha_q*(lnQ) + \alpha_f*(FIX) + \tfrac{1}{2}*\Sigma_i\Sigma_j\alpha_{ij}*(lnP_i*lnP_j) + \tfrac{1}{2}\Sigma_i\alpha_{iq}*(lnP_i*lnQ) + \Sigma_i\alpha_{if}*(lnP_i*FIX). \tag{4}$$

The α's are the parameters of the cost function with Q as output and FIX as fixed input (discussed below). The neoclassical production theory states that a production structure must be well behaved. For the cost function, this implies the following:

1) the cost function must be a strictly positive function for positive input prices and positive output levels;
2) the cost function is a smooth non-decreasing function in input prices;
3) the cost function is positively and linearly homogeneous in input prices, (when all prices double, the cost also doubles);
4) the cost function is concave in input prices; and
5) the cost function is a continuous function of prices given that the first and second partial derivatives exist.

When the above conditions are satisfied by a cost function, it is said to be well behaved. Translating the above conditions to application means the following set of requirements need to be imposed on the cost function.

i) **Adding up condition**: sum of input costs must equal total cost.

ii) **Cournot's aggregation**: producers can reallocate their input expenditures when input prices change but cannot violate the adding up condition.

iii) **Engel's aggregation**: for a given budget allocation, expenditure will not violate the adding up condition.

iv) **Symmetry**: the symmetry condition implies that the cross second order partial derivative is equal.

The above conditions can be readily translated in the context of the translog cost function and requires some a *priori* restrictions on the parameters of the cost function (Equation 5). The restrictions are defined in Eq 6 as follows:

$$\Sigma\alpha_i = 1; \Sigma_i\alpha_{ij} = \Sigma_j\alpha_{ji} = 0; \Sigma_i\Sigma_j\alpha_{ij} = 0 ; \alpha_{ij} = \alpha_{ji}; \Sigma_i\alpha_{iq} = 0; \Sigma_i\alpha_{if} = 0. \quad (5)$$

It is worth summarising some essential merits of this function in relation to the more common Cobb-Douglas and the CES functions. The translog function is a member of the family of the so-called flexible function forms. What this means is that these flexible functional forms have enough parameters which allow the determination of many economic characteristics of the production structure under investigation. The elasticities of substitution are not a *priori* restricted and can be estimated for each sample observation. Also, economies of scale are not restricted a *priori* and can be evaluated from the estimated parameters of the function. In other words, the flexible functional forms are unrestricted and different production structures such as the homogeneous production structure or the Cobb Douglas structure can be tested for using these flexible functional forms.

The duality principal or Shepherd's lemma, is crucial to the estimation of production attributes from well-behaved cost functions. The factor demand equations (in terms of factor input cost share) can be derived from this lemma and it can be written as follows for the translog cost function(Equation 6):

$$\delta lnC/\delta lnP_i = (P_i{*}W_i)/C = \alpha_i + \Sigma_i\alpha ij(lnP_j) + \alpha iq(lnQ) + \alpha if(FIX). \quad (6)$$

where W_i stands for the quantity of i-the input factor. The term $(P_i{*}W_i)/C = S_i$ gives i-the factor's input budget share. For the translog cost function, the Allen-Uzawa partial elasticities of substitution and price elasticities of demand are given by Eq. 7.

Substitution elasticities

$$\sigma_{ii} = \{S_{i2}\text{-}S_i + \alpha_{ii}\}/S_{i2}$$

$$\sigma_{ij} = \{S_i{*}S_j + \alpha_{ij}\}/(S_i{*}S_j) \quad (7)$$

Price elasticities

$$\epsilon_{ii} = \sigma_{ii}{*}S_i$$

$$\epsilon_{ij} = \sigma_{ij} = \{S_i{*}S_j + \alpha_{ij}\} \text{ for all i, j and i not equal to j}$$

The Demand for Energy Inputs

We will now discuss the derivation of the energy sub-model. The manufacturing sector can choose among a number of energy types, namely, wood (W), diesel (D), coal (C), kerosene (K), and electricity (E). Imposing weak separability in energy as discussed above, the sub-translog cost function for the energy sub-model can be derived as follows:

$$lnC_e = \beta_0 + \Sigma\beta_i{*}(lnR_i) + \tfrac{1}{2}{*}\Sigma_i\Sigma_j\beta_{ij}{*}(lnR_i{*}lnR_j). \quad (8)$$

where

C_e is the energy cost borne by the firm. The R's here represent the unit price of each energy type (wood, diesel, coal, kerosene, and electricity). The share equations derived above for the manufacturing sector can also be derived for the energy cost function as well as the substitution and factor demand elasticities. All the restrictions (Eqs 5) are also imposed on the energy sub-model.