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**Glacier volume  
response time**

S. C. B. Raper and  
R. J. Braithwaite

# Glacier volume response time and its links to climate and topography based on a conceptual model of glacier hypsometry

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## Abstract

Glacier volume response time is a measure of the time taken for a glacier to adjust its geometry to a climate change. It is currently believed that the volume response time is given approximately by the ratio of glacier thickness to ablation at the glacier terminus. We propose a new conceptual model of glacier hypsometry (area-altitude relation) and derive the volume response time where climatic and topographic parameters are separated. The former is expressed by mass balance gradients which we derive from glacier-climate modelling and the latter are quantified with data from the World Glacier Inventory. Aside from the well-known scaling relation between glacier volume and area, we establish a new scaling relation between glacier altitude range and area, and evaluate it for seven regions. The presence of this scaling parameter in our response time formula accounts for the mass balance elevation feedback and leads to longer response times than given by the simple ratio of glacier thickness to ablation. Volume response times range from decades to thousands of years for glaciers in maritime (wet-warm) and continental (dry-cold) climates, respectively. The combined effect of volume-area and altitude-area scaling relations is such that volume response time can increase with glacier area (Axel Heiberg Island and Svalbard), hardly change (Northern Scandinavia, Southern Norway and the Alps) or even get smaller (The Caucasus and New Zealand).

## 1 Introduction

Global warming is causing increased melt of grounded ice and contributing to sea level rise (Meier, 1984; IPCC, 2007). For projections of future sea-level rise, there are two stages in the modelling of glacier response to climate change (Warrick and Oerlemans, 1990). Firstly, to calculate changes in mass balance over present glacier areas (static response) and secondly to account for the changing glacier area and volume (dynamic response) that result from the mass balance changes. Earlier projections of

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sea-level only took account of the static response, e.g. as modelled by Oerlemans and Fortuin (1992) and Oerlemans (1993), while more recent assessments also include attempts to account for dynamic response (Van de Wal and Wild, 2001; Raper and Braithwaite, 2006).

5 The classic approach to changes in glacier volume as a result of climate forcing is to solve partial differential equations for glacier dynamics and thermodynamics. This can be done analytically for simplified glacier geometry (Nye, 1963) but will generally require numerical methods. Numerical solutions of differential equations are quite feasible for ice sheet modelling (Huybrechts and Payne, 1996) but difficult to apply to the  
10 several hundred thousand mountain glaciers and ice caps that contribute to rising sea level. For example, Oerlemans et al. (1998) apply dynamic flow models to 12 glaciers and say “No straightforward relationship between glacier size and fractional change of ice volume emerges for any given climatic scenario. The hypsometry of individual glaciers and ice caps plays an important role in their response, thus making it difficult to  
15 generalize results”. Dynamic flow modelling of more individual glaciers may eventually contribute to a better overall view of sea level rise from glaciers but several authors have proposed the alternative use of more conceptual models with volume-area scaling (Van de Wal and Wild, 2001; Raper and Braithwaite, 2006) that can be applied to a whole spectrum of glacier dimensions.

20 Glacier volume response time is a measure of the time taken for a glacier to adjust its geometry to a climate change (Jóhannesson et al., 1989a; Oerlemans, 2001) and is implicit in the solution of numerical models, e.g. the response to a step change in mass balance (Nye, 1963). However, a number of authors have sought to express volume response time analytically as a relatively simple function of climate and glacier  
25 geometry (Jóhannesson et al., 1989a; Raper et al., 1996; Bahr et al., 1998; Pfeffer et al., 1998; Harrison et al., 2001; Oerlemans, 2001). Such functions should allow us to estimate characteristic response times for different glacier regions from their climatic and topographic settings.

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The response time  $\tau$  according to Jóhannesson et al. (1989a) is given in Eq. (1):

$$\tau = \frac{H}{-b_t} \quad (1)$$

Where “ $H$  is a thickness scale of the glacier and  $-b_t$  is the scale of the ablation along its terminus” in the words of Jóhannesson et al. (1989a). Paterson (1994, p. 320) applies Eq. (1) to estimate response times for three classes of glacier: glaciers in temperate maritime climate, ice caps in arctic Canada and for the Greenland ice sheet. The resulting response times are respectively 15–60, 250–1000 and 3000 a. The short response time for the first class (with smallest area?) is noteworthy because a theoretical analysis by Nye (1963) concluded that alpine glaciers have response times of several hundred years. Paterson (1994, 320–321) cites the classic graph of percentage advancing/retreating glaciers in the Alps over the past century as evidence for quite short response times for Alpine glaciers, i.e. less than 20 a. Jóhannesson et al. (1989b) extend the approach of Jóhannesson et al. (1989a) by analysing the apparent discrepancy in response times between their formula, i.e. Eq. (1) in our paper, and expressions derived by Nye (1963) from kinematic wave theory. Apparently “... using kinematic wave theory involves difficult problems concerning details of behaviour at the terminus” (Jóhannesson et al., 1989b).

Analytical formulations of glacier response time depend upon conceptual models of the glacier that should be realistic but simple enough to yield an analytical solution. Callendar (1950 and 1951) was an early pioneer of conceptual models in glaciology and was able to explain some aspects of glacier behaviour with “back of the envelope” calculations. However, there can be a “law of unintended consequences” such that a simplification in the model (introduced to make calculations easier) excludes glacier behaviour that is relevant to the real world. For example, Jóhannesson et al. (1989a) admit that their approach neglects the positive feedback that arises when an initial mass balance perturbation causes a change in glacier surface altitude such that the mass balance perturbation changes. We show that this problem is inherent in the assumptions that lead to the use of the mass balance, or ablation, at the glacier terminus.

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The thickness scale in Eq. (1) is loosely defined as Jóhannesson et al. (1989a) emphasise their approach is only intended to give order of magnitude estimates and Paterson (1994, p. 320) says the distinction “. . . between the mean and maximum ice thickness is insignificant at this order of precision”. In addition, the differing effects of climate and glacier geometry in Eq. (1) are difficult to separate. For example,  $H$  in Eq. (1) is clearly a geometrical property of the glacier but  $b_t$  combines climatic and geometrical properties. We therefore propose an improved derivation of the volume response time in Raper et al. (1996), where the climate and geometric parameters are separated, and the latter are clearly defined so that we can quantify them with data from the World Glacier Inventory ([http://nsidc.org/data/glacier\\_inventory](http://nsidc.org/data/glacier_inventory)). Our conceptual model is based upon the simplified model of glacier hypsometry (area-altitude relation) used by Raper and Braithwaite (2006), while Jóhannesson et al. (1989a) and Oerlemans (2001, Chapter 8) use cross-sections (altitude versus downstream distance) of uniform width. Our model treats the mass balance of the whole glacier rather than mass balance near the terminus. We hope that our work follows the philosophy that it is “better to think exactly with simplified ideas than to reason inexactly with complex ones” (Nye, 1948).

## 2 Our conceptual model

For a glacier in equilibrium with climate the mean specific mass balance, i.e. the area-averaged balance for the whole glacier (Anonymous, 1969) is zero. Its equilibrium line altitude (ELA) is  $ELA_0$  and its dimensions are constant with time (see Table 1 for model notation). The glacier has area  $A_0$  and volume  $V_0$ . The mass balance is then suddenly perturbed by  $\Delta b$  applied over the whole area of the glacier and the ELA instantly rises to  $ELA_1$ , and the dimensions of the glacier slowly change to bring the glacier back into a new equilibrium state with area  $A_1$  and volume  $V_1$ .

The conceptual model is sketched in Fig. 1. The change in mass balance of the whole glacier is represented in Fig. 1 by a shift in the balance-altitude relation which

we assume is linear over the whole glacier. The new ELA of the glacier,  $ELA_1$ , depends upon the magnitude of the mass balance perturbation and the balance gradient  $k$ , such that:

$$ELA_1 = ELA_0 + \frac{\Delta b}{k} \quad (2)$$

Figure 1 is drawn for a typical alpine glacier where a temperature rise of +1 K throughout the whole year causes a mass balance change of  $-0.66 \text{ m water } a^{-1}$  and raises the ELA by 160 m (values from the model calculations for Braithwaite and Raper, 2007). The change in mass balance for a 1 K temperature rise is termed the temperature sensitivity of mass balance (Oerlemans and Fortuin, 1992; Braithwaite and Zhang, 1999; de Woul and Hock, 2005). Equation (2) could be applied to a mass balance change due to precipitation change and we would then use the precipitation sensitivity of mass balance. For the present paper we abbreviate “temperature sensitivity of mass balance” to ‘mass balance sensitivity’ because we are only dealing with temperature changes.

Our conceptual model is simple but not unrealistic:

1. Balance gradients are not generally constant with altitude on real glaciers, reflecting effects of precipitation variations as well as wind drifting and avalanches. Balance gradients are also greater in the ablation area and smaller in the accumulation area (Furbish and Andrews, 1984; Braithwaite and Raper, 2007). We assume a constant balance gradient here to derive an analytical solution for glacier volume change but for numerical modelling we can use the degree-day model to calculate non-linear balance-altitude relations, e.g. as in Braithwaite and Raper (2007).
2. Figure 1 shows triangular area-altitude distributions that are symmetrical about the ELA (Raper and Braithwaite, 2006) and we also assume that the glacier retains a symmetrical altitude distribution as it shrinks. We return to these points later in the paper with data from real glaciers to show these are reasonable first approximations.

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3. The apex of the triangle in Fig. 1 is the median altitude of the glacier, dividing the glacier area into equal halves, and this coincides with the ELA when the glacier is in equilibrium (Meier and Post, 1962; Braithwaite and Müller, 1980). For a symmetric area-altitude distribution, the median altitude is also equal to the area-weighted mean altitude of the glacier (Kurowski, 1893).
4. For a linear balance-gradient, the specific mass balance at the median altitude (also the mean altitude) is equal to the mean specific mass balance (Kurowski, 1893; Braithwaite and Müller, 1980).
5. The assumption of linear balance-gradient is not as restrictive as it may first appear because glacier areas are generally largest near the median altitude, where there is maximum ice flux, and smallest at the top and bottom of the glacier where the linear assumption is most likely to be violated. The  $k$  parameter in Eq. (2) should therefore be regarded as representing the balance gradient near the ELA, i.e. the activity index of Meier (1962) or the energy of glacierization of Shumsky (1947).

We measure all heights downwards from the top of the glacier which is assumed fixed when climate changes. This is a reasonable assumption for valley and mountain glaciers whose tops are constrained by hard rock topography but does not apply to ice caps since the maximum height of an ice cap changes with the ice cap thickness. The approach used in this paper does not therefore apply to ice caps. As the maximum altitude of the glacier is fixed, shrinkage of the glacier involves a rise in the minimum elevation, or a reduction in the altitude range  $R$  between maximum and minimum altitudes of the glacier. As the glacier's area and volume shrink towards the new equilibrium state, the half-range ( $R/2$ ) moves from  $R_0/2$  at the old ELA to  $R_1/2$  at the new ELA. The mass balance  $\bar{b}$  of the whole glacier at any time is then equal to:

$$\bar{b} = k \frac{(R_1 - R)}{2} \quad (3)$$

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Equation (3) follows from Eq. (2) when the initial mass balance perturbation,  $\Delta b$ , and initial altitude range  $R_0$ , are replaced by the time-evolving variables  $\bar{b}$  and  $R$ . Measuring heights downwards,  $R$  is bigger than  $R_1$  and  $\bar{b}$  is therefore negative and tends to zero as  $R$  approaches  $R_1$ .

The mass balance at the glacier terminus (altitude  $R$  measured from the top of the glacier) at any time after the mass balance perturbation is given by

$$b_t = k \left[ \left( \frac{R_0}{2} \right) - R \right] + \Delta b \tag{4}$$

The mass balance at the terminus is therefore a function of both climate (balance gradient,  $k$ ) and glacier geometry (altitude range,  $R$ ) and changes until it achieves a new steady-state value for  $R=R_1$ . The value of  $R_1$  can be calculated from Eq. (2) by noting that  $ELA_1=R_1/2$  and  $ELA_0=R_0/2$  which gives:

$$R_1 = R_0 + 2 \left( \frac{\Delta b}{k} \right) \tag{5}$$

The total change in altitude of the terminus according to Eq. (5) is twice the change in ELA in Eq. (2). We return to the issue of the mass balance at the terminus at the end of this section and in a later section but start our analysis with the mean specific balance. The product of the glacier area  $A$  and mean specific balance, given by Eq. (3), gives the rate of change of glacier volume:

$$\frac{dV}{dt} = Ak \frac{(R_1 - R)}{2} \tag{6}$$

We treat ice dynamics implicitly by using geometric scaling following Chen and Ohmura (1990), Bahr et al. (1997), Raper et al. (1996, 2000), Van de Wal and Wild (2001), Raper and Braithwaite (2006) and Radic et al. (2007, 2008). We follow others in assuming that volume scales onto area as

$$V \propto A^Y \tag{7}$$

and we use a scaling relation between altitude range and area as implemented by Raper and Braithwaite (2006) so that

$$R \propto A^\eta. \quad (8)$$

Bahr et al. (1997) estimate the scaling index  $\gamma$  in Eq. (7) from depth-sounding observations on 144 glaciers and obtain an average of 1.36, which is in remarkable agreement with Chen and Ohmura (1990) who studied fewer glaciers. Bahr et al. (1997) also quote Russian authors for indices in the range 1.3 to 1.4. Paterson (1981, 1994) gives indices of 1.25 and 1.5, respectively, for theoretical profiles of ice caps and valley glaciers. The volume-area scaling of Bahr et al. (1997) has been criticised as it “correlates a statistical variable (area) with itself (area in volume)” (Haerberli et al., 2007; Lüthi et al., 2008) but we think it is a good example of an empirical relation with a sound physical background. The  $\gamma$  value is greater than unity because average glacier depth increases with increasing glacier area.

From Eqs. (7 and 8) expressions for  $A$  and  $R$  can be derived in terms of  $V$  and  $V_0$ , giving:

$$A = A_0 \left( \frac{V}{V_0} \right)^{\frac{1}{\gamma}} \quad (9)$$

$$R = R_0 \left( \frac{V}{V_0} \right)^{\frac{\eta}{\gamma}} \quad (10)$$

Substitution of Eqs. (9 and 10) into Eq. (6) gives a new form of the volume change equation:

$$\frac{dV}{dt} = A_0 \left( \frac{V}{V_0} \right)^{\frac{1}{\gamma}} k \left( \frac{R_0}{2} \right) \left[ \left( \frac{V_1}{V_0} \right)^{\frac{\eta}{\gamma}} - \left( \frac{V}{V_0} \right)^{\frac{\eta}{\gamma}} \right] \quad (11)$$

If we now define a new volume variable  $Y$  by:

$$Y = V^{\frac{\eta}{\gamma}} \quad (12)$$

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Then Eq. (11) may be written in more conventional linear-response form involving a time scale,  $\tau$ , as:

$$\frac{dY}{dt} = \frac{(Y_1 - Y)}{\tau} \quad (13)$$

Where  $Y_1$  and  $Y$  are  $V_1^{n/\gamma}$  and  $V^{n/\gamma}$ , respectively.  $Y$  refers to the current glacier volume while  $Y_1$  refers to the new equilibrium volume towards which the current volume is tending. The glacier volume response time,  $\tau$ , is defined as:

$$\tau = \left(\frac{\gamma}{\eta}\right) D_0 \left(\frac{2}{R_0}\right) \left(\frac{1}{k}\right) \left(\frac{V}{V_0}\right)^{\frac{(\gamma-1-\eta)}{\gamma}} \quad (14)$$

where the mean glacier depth  $D_0$  equals  $V_0/A_0$ . The last term determines how the response time changes non-linearly with changing volume. However, for a glacier in or near its reference state, i.e.  $V \approx V_0$  for small volume changes, the equation is simplified because the fifth term becomes unity:

$$\tau = \left(\frac{\gamma}{\eta}\right) D_0 \left(\frac{2}{R_0}\right) \left(\frac{1}{k}\right) \quad (15)$$

This is the form of the equation that we use later in the paper. However, it is useful to briefly compare our formulation with that of Jóhannesson et al. (1989a). A more in-depth comparison is made in a later section.

In Eq. (15) the first term ( $\gamma/\eta$ ) involves the scaling factors, the second term ( $D_0$ ) represents glacier geometry before perturbation of mass balance, and according to Eq. (4) the inverse of the third and fourth term  $k.(R_0/2)$  represents  $-b_t$  where  $b_t$  is the balance at the terminus before perturbation of mass balance. Our response time Eq. (15) can therefore be expressed by:

$$\tau = \left(\frac{\gamma}{\eta}\right) \left(\frac{D_0}{-b_t}\right) \quad (16)$$

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Comparing Eq. (16) with Eq. (1) from Jóhannesson et al. (1989a), shows that both equations involve a ratio of glacier depth to balance at the terminus but our equation for response time Eq. (16) involves an extra numerical factor of  $(\gamma/\eta)$ .

## 2.1 The effects of climate change

5 The effects of climate change on glacier volume are expressed by the mass balance sensitivity and the response time. The former defines the immediate change in mass balance caused by a particular change in temperature and the latter measures the speed at which the glacier volume adjusts to the climate change. The response time depends upon the mass balance gradient according to Eq. (15) as well as upon glacier  
10 dimensions.

We use mass balance sensitivities and gradients calculated with a degree-day model for seven regions (Braithwaite and Raper, 2007). The degree-day model is applied to the estimated ELA for half-degree latitude-longitude grid squares containing glaciers within each region. The estimated ELA for each grid square is the  
15 average of median altitude for all glaciers within the grid square. The regions (Table 2) were chosen for their good data coverage in the World Glacier Inventory ([http://nsidc.org/data/glacier\\_inventory](http://nsidc.org/data/glacier_inventory)). They include the five main glacial regions of Europe together with Axel Heiberg Island, Canada, and New Zealand, which were added to represent the extremes of cold/dry and warm/wet conditions.

20 Average and standard deviation of mass balance sensitivity and gradient are shown in Fig. 2 for the seven regions. The mass balance sensitivity and balance gradient both vary by about an order of magnitude. The mass balance model gives a very strong negative linear relationship between the mass balance sensitivity and the mass balance gradient ( $r=-1.00$ ) with low (negative) sensitivity and low gradient in dry-cold conditions and high (negative) sensitivity and high gradient in wet-warm conditions.  
25 The physical basis for the relationship is that the mass balance sensitivity is a change in the mass balance due to a temperature change whereas the mass balance gradient is a change in the mass balance due to an altitude change. Thus the relationship

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between mass balance sensitivity and mass balance gradient are to a large extent governed by the temperature lapse rate, see Kuhn (1989).

The link between the mass balance sensitivity and the glacier volume response time (through the mass balance gradient) has implications for long-term changes in glacier volume and global sea-level. It means that warm/wet glaciers with large mass balance sensitivity tend to have a small response time whereas cold/dry glaciers with small mass balance sensitivity tend to have a longer response time. This behaviour was noted by Raper et al. (2000) in sensitivity experiments. Thus temperature-sensitive glaciers show the most rapid response to climate change at present but may not be the most important contributors to sea level change in the long term.

### 3 The effects of topography

In this section, we analyse the World Glacier Inventory data to estimate the area vs altitude scaling index values appropriate for the seven regions with their particular topographic relief. The World Glacier Inventory data mostly originates from measurements taken in the third quarter of the 20th Century (~1950–1975). This was a time of relative glacier stability and the worldwide mass balance was generally smaller (closer to zero) than more recently (Kaser et al., 2006). First, however, we examine the validity of our assumption of triangular and symmetric area vs altitude distribution.

The World Glacier Inventory ([http://nsidc.org/data/glacier\\_inventory](http://nsidc.org/data/glacier_inventory)) is coded according to the instructions of Müller et al. (1977). There are data for a total of 14 387 glaciers covering a total area of 53 481 km<sup>2</sup> for the seven regions (Table 2). This represents less than 10% of all the mountain glaciers and ice caps on the globe, most of which are still not included in the World Glacier Inventory. For the present study, the required variables are area, maximum, minimum, and median elevation and primary classification for each glacier.

The instructions of Müller et al. (1977) actually refer to mean altitude but their definition clearly refers to the median altitude, see discussion at the end of Braithwaite

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and Müller (1980). We can then compare the median altitude (properly defined) with the mid-point altitude representing the average of maximum and minimum glacier altitudes (note that many authors incorrectly use the term median for the latter). The difference between median and mid-point altitude is a measure of the asymmetry of the area-altitude distribution. There are missing data for one or other altitude for many glaciers. For example, data are not available at all for median altitude for regions 1, 3 and 4 (Axel Heiberg Island, Northern Scandinavia and Southern Norway), and for region 6 (Caucasus) the listed median altitudes are identical to the mid-point altitudes. Glacier inventories for these areas were finished before the instructions of Müller et al. (1977) were available, thus accounting for the omission of median elevation (regions 1, 3 and 4) or its wrong definition (region 6).

The comparison between median and mid-point altitudes could be made for 6976 glaciers in three of the seven regions. However, our conceptual model (Fig. 1) does not apply to “ice caps” and to “outlet glaciers” and we exclude them from the data set, i.e. glaciers with primary classification equal to 3 and 4 according to Müller et al. (1977). Median altitude can then be compared with mid-point altitude for 6831 glaciers. On average there is a remarkably good agreement, with very little difference (mean and standard deviation) between the two altitudes compared with the altitude range of glaciers (Table 3). There is also a high correlation and nearly 1:1 relation between the two altitudes (Fig. 3). The assumption of a symmetrical altitude-area distribution in our conceptual model is therefore very sound within a few decametres, e.g. the root-mean-square error of the regression line in Fig. 4 is only  $\pm 61$  m. We believe that glacier area-altitude distributions may actually be somewhat asymmetric during periods of advance or retreat but the overall effect (Table 3) is small: presumably the sample of 6831 glaciers contains a mixture of retreating, stationary and advancing glaciers.

With a view to modelling the hundreds of thousands of the world's glaciers for the sea level contribution of glacier melt, we are interested in identifying regional differences in glacier response. Oerlemans (2007) has shown that glaciers adjacent to each other

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can differ in their response times due to different geometry. This can be thought of as noise superimposed on a common signal of glacier response in a region determined by the regional climate and topography. In the empirical derivation of regional scaling indices that follows, we assume that a typical glacier growing or shrinking in a region will in general still have its geometrical dimensions governed by that region's topographical relief. According to the scaling relation (Eq. 8), altitude range is linked to area and we estimate the scaling parameter  $\eta$  by regression analysis of data from the glacier inventory. There are two ways that this could be done: nonlinear regression with the original data for range and area or linear regression with logarithms of range and area. Results are fairly similar but we think the first approach gives better fit to the larger glaciers. Statistical results for the nonlinear regression equation  $R \propto A^\eta$  are given in Table 4 for each of the seven regions. The regression lines from the nonlinear analysis are re-plotted as straight lines in log-log space in Fig. 4 to illustrate the overall fit of the data.

The estimated values of  $\eta$  in Table 4 are rather similar (0.3 to 0.4) for six of the regions but relatively low (0.07) for Svalbard. The latter is clearly anomalous and must be partly due to the omission of smaller glaciers ( $<1 \text{ km}^2$ ) from the glacier inventory compared with the other regions in Fig. 4. As well as supplying us with estimates of  $\eta$  for our response time Eq. (15), the regression analysis allows us to identify typical altitude ranges for a selection of glacier areas to explore response times for generic glaciers (see below).

#### 4 The volume response time for seven regions

Using the formula for volume response time  $\tau$  in Eq. (15) we make calculations for some generic glaciers to illustrate the effects of differing geometry and climate regime (Table 5). Results are calculated for glacier areas of 1, 10, 50  $\text{km}^2$ , respectively, with corresponding scaled glacier depths of 28, 65 and 117 m, respectively (for  $\gamma=1.36$ ). The altitude ranges for the generic glaciers are given in Table 5, using the  $\eta$  value for

the region in question.

Some work on response time has suggested it should increase with glacier size, e.g. Table 13.1 in Paterson (1994), but Bahr et al. (1998) found that under certain conditions the response time will decrease as size increases. In seeming confirmation of this, Oerlemans et al. (1998) found “No straightforward relationship between glacier size and fractional change of ice volume”. In our Table 5 response time does increase with area for regions 1 and 2 (Axel Heiberg Island and Svalbard), hardly increases in regions 3, 4 and 5 (Northern Scandinavia, Southern Norway and the Alps), and decreases in regions 6 and 7 (Caucasus and New Zealand). The reasons for this can be understood by looking at the detailed breakdown in Table 5.

The reciprocal of balance gradient in line 4 of Table 5 has the units of time (a) and expresses the basic effect of climate on response time with a range of  $10^2$  to  $10^3$  a, from the warm-wet (maritime) climate of New Zealand to the cold-dry (continental) climate of Axel Heiberg Island. Regions 1 and 2 both represent arctic islands but region 1 is obviously more continental than region 2. Balance gradients in regions 3, 4, 5 and 6 are quite similar while there is a further jump to the very maritime climate of region 7. This order of magnitude climate influence on the glacier volume response time is then modified by geometric factors.

With reference to Table 5, the geometric factor  $R_0$  differs by a factor of 3 between regions 1–4 and regions 5–7 and has a corresponding influence on the volume response times. The geometric factor that determines how the volume response time depends on glacier size is the depth-range ratio,  $D_0/R_0$  (the middle terms of Eq. 15), which scales as  $A_0^{\gamma-1-\eta}$ . In Table 5, the index  $\gamma - 1 - \eta$  is either small positive or small negative, aside from anomalous results for region 2 (Svalbard). Examination of Table 5 shows that for the regions with negative index  $\gamma - 1 - \eta$ ,  $D_0/R_0$  decreases with increasing glacier area, leading to shorter response times for larger glaciers in regions 6 and 7.

We suggest that the larger values of  $\eta$  in regions 6 and 7 and the associated larger altitudinal ranges for relatively big glaciers (for example of area  $10 \text{ km}^2$ , which is well within the data range for all seven regions) are related to regional topography. High

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values of  $\eta$  indicate rapidly increasing altitudinal range with increasing glacier area and may be associated with steeper terrain as suggested by Raper and Braithwaite (2005, 2006). We hope to explore this issue in a future study using data from other regions with even greater altitude ranges.

We are not able to differentiate between different regions in our volume-area scaling and we simply use the scaling factor  $\gamma=1.36$  from Bahr et al. (1997). This means we have to assume a single glacier depth for a particular area in Table 5 but we speculate that a regionally-specific glacier depth would further increase the range of response times shown in Table 5. The reasoning is that glaciers in regions with high topographic relief, with large altitude ranges, are likely to be faster flowing and thinner (smaller  $D_0$ ) than those in relatively flat terrain, thus further reducing already short response times.

## 5 An alternative approach

We start by returning to the definition of volume response time  $\tau$  in Jóhannesson et al. (1989a):

$$\tau = \frac{\partial V}{\partial B} \quad (17)$$

Where  $\partial V$  is the difference in the steady state volume of the glacier before and after the mass balance perturbation, and  $\partial B$  is the integral of the mass balance perturbation over the initial area of the glacier. Jóhannesson et al. (1989a) then give expressions for these terms for some simplified glacier geometries. Their crucial assumption is that the initial mass balance perturbation of the whole glacier is balanced in the new steady state by an area change at the glacier terminus over which the mean specific mass balance is the initial mass balance at the terminus  $b_t$ . We, however, can apply our conceptual model to Eq. (17) as follows:

$$\partial V = (V_1 - V_0) \quad (18)$$

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$$\partial B = A_0 k \frac{(R_1 - R_0)}{2} \quad (19)$$

Where  $\partial B$  refers to the volumetric balance obtained by multiplying specific balances from Eq. (3) by glacier area.

Substituting the volume-area scaling Eq. (9) into Eq. (18) gives:

$$\partial V = V_0 \left[ \left( \frac{A_1}{A_0} \right)^\gamma - 1 \right] \quad (20)$$

For relatively small area changes, we can use a truncated McLaurin series to replace  $(A_1/A_0)^\gamma$  with  $1 - \gamma [1 - (A_1/A_0)]$ . Substituting this linear approximation back into Eq. (20), with re-arrangement and expressing  $V_0/A_0$  as the mean depth of the glacier  $D_0$  gives:

$$\partial V = D_0 \gamma [A_1 - A_0] \quad (21)$$

In a similar way:

$$\partial B = k \left( \frac{R_0}{2} \right) \eta [A_1 - A_0] \quad (22)$$

Dividing Eq. (20 by 21) then gives us

$$\tau = \frac{[D_0 \gamma]}{\left[ k \left( \frac{R_0}{2} \right) \eta \right]} \quad (23)$$

This is identical to our original derivation in Eq. (15).

From an inspection of Eqs. (10 and 11) in Jóhannesson et al. (1989a), and translating into our notation, his thickness scale  $H$  is given by:

$$H = \left( \frac{dV}{dA} \right) \left( \frac{A_0}{V_0} \right) D_0 \quad (24)$$

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Where  $D_0$  is the mean depth of the glacier. Jóhannesson et al. (1989a) evaluate the expression  $(dV/dA) (A_0/V_0)$  in terms of analytical solutions for simplified glacier geometry. We now evaluate this expression using the volume-area scaling method that was not available in its present form in 1989. Expressing  $V$  as  $V_0 (A/A_0)^\gamma$  and differentiating with respect to  $A$ , we find  $(dV/dA) (A_0/V_0) = \gamma$ . Therefore:

$$H = \gamma D_0 \quad (25)$$

Substitution of Eq. (25) back into Eq. (1) gives the response time of Jóhannesson et al. (1989a) as:

$$\tau_{\text{JRW}} = \frac{\gamma D_0}{-b_t} \quad (26)$$

Where  $\tau_{\text{JRW}}$  denotes the response time according to Jóhannesson et al. (1989a).

Comparing Eq. (26) with (16) shows that:

$$\tau_{\text{RB}} = \frac{\tau_{\text{JRW}}}{\eta} \quad (27)$$

Where  $\tau_{\text{RB}}$  is the response time according to the present authors Raper and Braithwaite.

## 6 Discussion

Using a mid estimate of  $\eta$  of 0.36, our response time  $\tau_{\text{RB}}$  is about 2.8 ( $1/0.36=2.78$ ) times longer than  $\tau_{\text{JRW}}$ , the response time of Jóhannesson et al. (1989a). An  $\eta$  value of unity ( $R \propto A$ ) is implicit in the glacier of unit or uniform width with all area changes at the terminus assumed by Jóhannesson et al. (1989a). Radic et al. (2007) also used a glacier of uniform width (and slope) in a comparison of a flow line model with a scaling model.

As a simple experiment with our conceptual model for triangular area-altitude distribution (Fig. 1), we calculated altitude range  $R_1$  and the corresponding area-altitude

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distribution for different values of  $\eta$  when the area shrinks from  $A_0$  to  $A_1$ . For  $\eta > 0.5$ , a large reduction in range to  $R_1$  would result in the dashed line above  $R_1/2$ , in Fig. 1, being above the solid line joining the top of the glacier to  $R_0/2$ . This is equivalent to a raising of the glacier surface due to thickening in the accumulation area as the total area is reduced, which is clearly unrealistic. For  $\eta = 0.5$ , the solid and dashed lines above  $R_1/2$  in Fig. 1 would coincide, implying no change in area or surface altitude above the new ELA for a change in area from  $A_0$  to the smaller  $A_1$ . The geometry changes that occur for  $\eta < 0.5$  as shown in Fig. 1 are consistent with a lowering of the glacier surface due to thinning in both accumulation and ablation areas, when going from  $A_0$  to the smaller  $A_1$ . Thus the formulation of the model in the vertical area-altitude dimension allows the mass balance-elevation feedback associated with both the area reduction and lowering of the glacier surface to be accounted for. Our empirically derived values of  $\eta$  of 0.3–0.4 are consistent with the geometric expectations that  $\eta < 0.5$ . In accord with our results, Oerlemans (2001, Table 8.3) found that, when applied to four real glaciers, his flow line model gave glacier volume response times that are longer than implied by the Jóhannesson et al. (1989a) formula. Jóhannesson et al. (1989a, b) do realise that their crucial assumption (see above) means that balance-elevation feedback is not accounted for and that their response time estimates may therefore be too short. Harrison et al. (2001) attempt to correct this by including a term that explicitly addresses the balance-elevation feedback, but they still assume that all the area change is close to the terminus. Their treatment cannot account for area-altitude changes due to thickening/thinning of the glacier far removed from the region of the terminus, i.e. in the accumulation area.

Our modelling of both mass balance and glacier area in the vertical dimension means that the mass balance elevation feedback resulting from both changes in total area and changes in height of the glacier surface are accounted for. The situation is not so clear when modelling is orientated along the glacier length (or horizontally) as in Radic et al. (2008), who say “The scaling methods applied here include feedback due to changes in area-elevation distribution, but lack the mass-balance/thickness feedback,

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i.e. the glacier area may change, but the thickness along the glacier profile does not". We find this statement puzzling since logically changes in the area-elevation distribution should reflect changes in glacier thickness.

Due to lack of data we are not able to estimate regionally specific values for  $\gamma$ , though we might expect lower/higher values of  $\gamma$  to be associated with lower/higher topographic relief. This issue should be resolved by extending the dataset by depth-sounding or other means (Farinotti et al., 2008) of estimating glacier volume for many more glaciers, especially in regions with high topographic relief. We note that changes in sub-glacial hydrology or in thermal conditions at the glacier bed may also have an effect on  $\gamma$ .

## 7 Conclusions

We propose a new derivation of the volume response time where the climate and geometric parameters are separated. The volume response time depends directly upon the mean glacier thickness, and indirectly on glacier altitude range and vertical mass-balance gradient. Our formula can be reduced to the well-known ratio of glacier thickness to mass balance at the glacier terminus (Jóhannesson et al., 1989a) but involves an extra numerical factor. This factor increases our volume response time by a factor of about 2.8 compared with that of Jóhannesson et al. (1989a) and reflects the mass balance elevation feedback.

Because volume response time depends upon vertical mass balance gradient, warm/wet glaciers tend to have shorter response times and cold/dry glaciers tend to have longer response times.

Our model is derived for glaciers whose area distribution is assumed to be symmetrical around the equilibrium line altitude (ELA). It is long known that in steady-state the ELA is approximately equal to median glacier altitude and we now confirm that the mid-point altitude is approximately equal to median altitude.

The scaling parameter between glacier-altitude range and area varies between re-

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gions such that volume response time can increase with area (Axel Heiberg Island and Svalbard), hardly change (Northern Scandinavia, Southern Norway and the Alps), or even get smaller (Caucasus and New Zealand).

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**Table 1.** Description of model parameters.

Parameter	Description
$\Delta b$	Change in mass balance (m w.e. a <sup>-1</sup> )
$\gamma$	Volume-area scaling parameter (dimensionless)
$\eta$	Altitude range-area scaling parameter (dimensionless)
$\tau$	Response time of glacier (a)
$\tau_{JRW}$	Response time from Jóhannesson et al. (1989a)(a)
$\tau_{RB}$	Response time according to present study (a)
$A$	Area of glacier at any time (km <sup>2</sup> )
$A_0$	Initial area of glacier (km <sup>2</sup> )
$A_1$	Final area of glacier (km <sup>2</sup> )
$\bar{b}$	Mean specific balance of glacier (m w.e. a <sup>-1</sup> )
$b_t$	Specific balance at terminus (m w.e. a <sup>-1</sup> )
$D_0$	Mean thickness of glacier before change of mass balance (m)
ELA	ELA of glacier at any time (m)
ELA <sub>0</sub>	ELA before change of mass balance (m)
ELA <sub>1</sub>	ELA after change of mass balance (m)
$H$	Thickness scale of glacier (m)
$k$	Balance gradient (m w.e. m <sup>-1</sup> )
$R$	Altitude range of glacier at any time (m)
$R_1$	Final altitude range (m)
$R_0$	Initial altitude range (m)
$V$	Volume of glacier at any time (km <sup>3</sup> )
$V_0$	Initial volume of glacier (km <sup>3</sup> )
$V_1$	Final volume of glacier (km <sup>3</sup> )

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Region	Lat/Long	Mid-point altitude (m a.s.l.)	Glacier area (km <sup>2</sup> )	Number of glaciers
Axel Heiberg Island	78–81° N 88–97° W	50–2200	11 691	1094
Svalbard	77–81° N 11–32° E	110–955	33 121	893
Northern Scandinavia	65–70° N 13–23° E	290–2080	1441	1487
Southern Norway	60–63° N 5–9° E	600–2060	1617	921
The Alps	44–47° N 6–14° E	1750–4060	3056	5331
The Caucasus	41–44° N 40–48° E	1960–4990	1398	1524
New Zealand	39–46° S 167–176° W	1030–2770	1158	3137
<b>TOTAL</b>			<b>53 481</b>	<b>14 387</b>

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Concept	Mean and standard deviation
Difference between median and mid-point altitudes	$-3 \pm 62$ m
Altitude range	$417 \pm 340$ m
Difference/Altitude	$-1 \pm 12\%$

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**Table 4.** Parameter results from non-linear regression between altitude range  $R$  and area  $A$  for seven regions.

Region	$R \pm 1$ SD (m) (range for area=1 km <sup>2</sup> )	$\eta \pm 1$ SD	Correlation coefficient	Glaciers
Axel Heiberg Island	366±8	0.30±0.008	0.67	777
Svalbard	562±27	0.07±0.014	0.28	271
N Scandinavia	387±0	0.34±0.000	0.72	1307
S Norway	363±5	0.31±0.008	0.72	823
The Alps	710±4	0.35±0.004	0.73	5291
The Caucasus	730±8	0.40±0.007	0.73	1192
New Zealand	729±5	0.37±0.003	0.78	2,744

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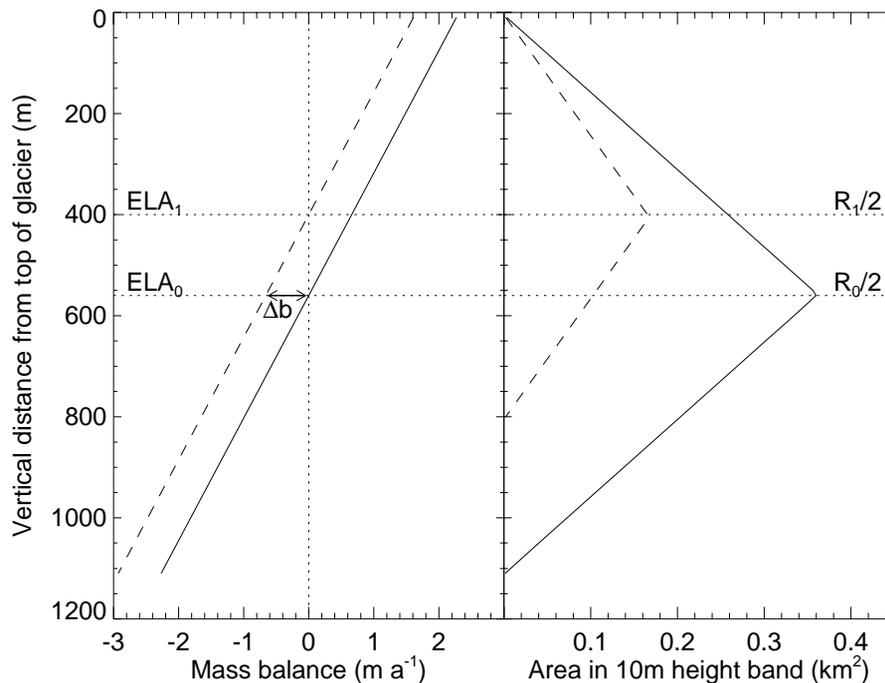
**Table 5.** Parameters and variables needed to calculate response times for three different values of glacier area in seven different regions. Regions are (1) Axel Heiberg Island, (2) Svalbard, (3) Northern Scandinavia, (4) Southern Norway, (5) Alps, (6) Caucasus and (7) New Zealand.

REGION			1	2	3	4	5	6	7
Variable	Units	Symbol							
Volume-area parameter		$\gamma$	1.36	1.36	1.36	1.36	1.36	1.36	1.36
Range-area parameter		$\eta$	0.30	0.07	0.34	0.31	0.35	0.40	0.37
Thickness-range parameter		$\gamma - 1 - \eta$	0.06	0.29	0.02	0.05	0.01	-0.04	-0.01
Reciprocal of balance-gradient	$a$	$1/k$	1111	455	250	204	233	208	108
Reference depth for area 1 km <sup>2</sup>	m	$D_0$	28	28	28	28	28	28	28
Reference depth for area 10 km <sup>2</sup>	m	$D_0$	65	65	65	65	65	65	65
Reference depth for area 50 km <sup>2</sup>	m	$D_0$	117	117	117	117	117	117	117
Altitude range for area 1 km <sup>2</sup>	m	$R_0$	366	562	388	363	710	730	729
Altitude range for area 10 km <sup>2</sup>	m	$R_0$	723	664	841	744	1592	1853	1727
Altitude range for area 50 km <sup>2</sup>	m	$R_0$	1165	747	1445	1227	2799	3552	3155
Thickness/Range for area 1 km <sup>2</sup>	–	$D_0/R_0$	0.077	0.052	0.072	0.077	0.039	0.038	0.038
Thickness/Range for area 10 km <sup>2</sup>	–	$D_0/R_0$	0.090	0.098	0.077	0.087	0.041	0.035	0.038
Thickness/Range for area 50 km <sup>2</sup>	–	$D_0/R_0$	0.100	0.157	0.081	0.095	0.042	0.033	0.037
Response time for area 1 km <sup>2</sup>	$a$	$\tau$	771	880	144	138	71	54	30
Response time for area 10 km <sup>2</sup>	$a$	$\tau$	906	1729	155	156	74	50	30
Response time for area 50 km <sup>2</sup>	$a$	$\tau$	1012	2766	162	171	76	47	29

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**Fig. 1.** Illustration of conceptual model of mass balance change by vertical displacement of a linear relation between mass balance and altitude. Solid lines show the reference climate state 0, dashed lines show the new perturbed climate equilibrium state 1. The area altitude distribution is assumed to be symmetrical and triangular.

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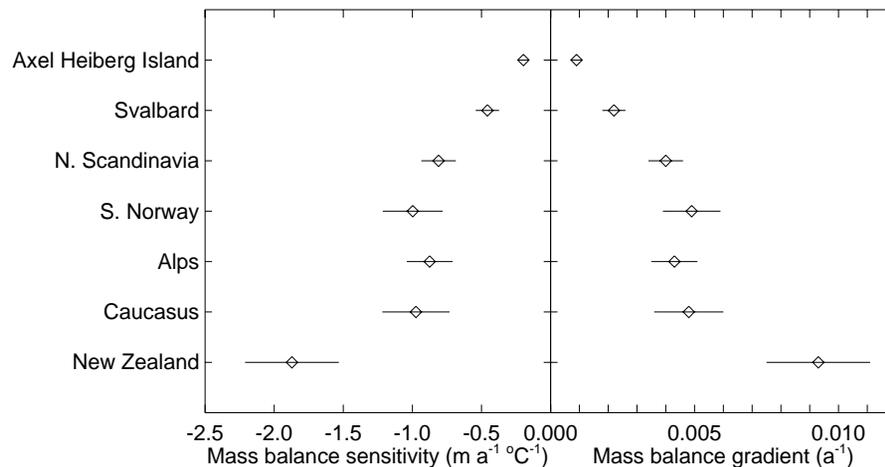
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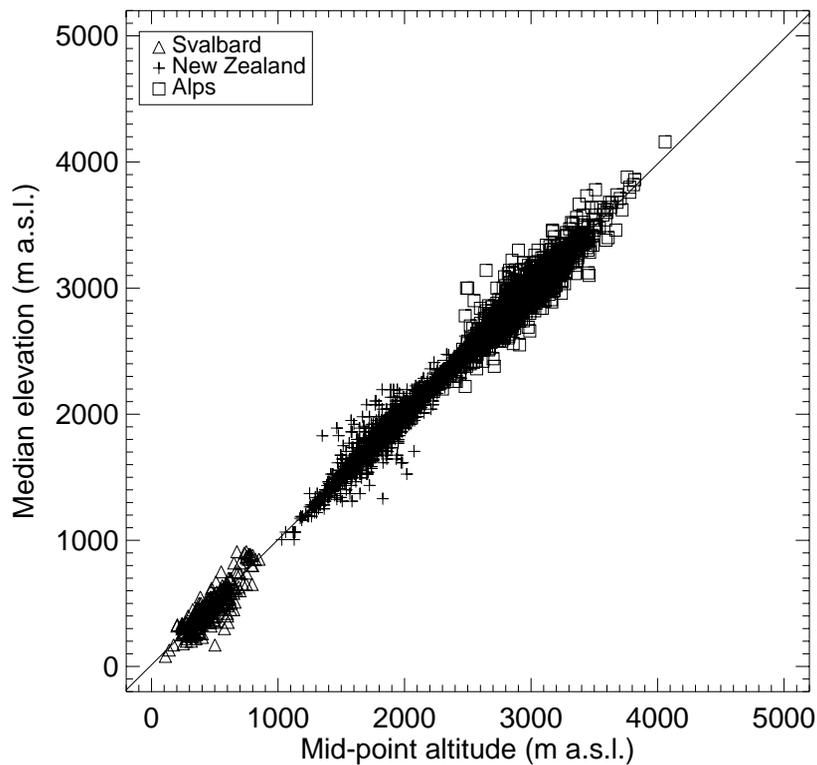
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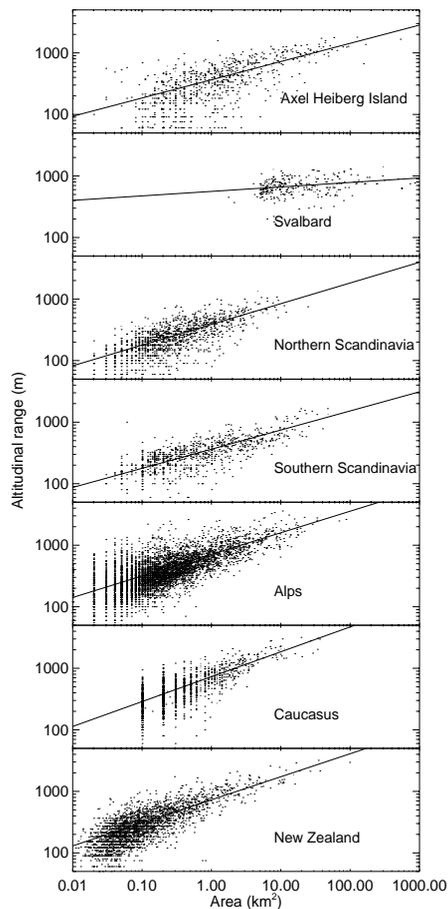
**Fig. 2.** Mass balance sensitivity and mass balance gradient represented by the activity index for the seven regions. Error bars are for  $\pm 1$  standard deviation.

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**Fig. 3.** Median elevation plotted against mid-point elevation for 6831 glaciers in three regions (excluding ice caps).

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**Fig. 4.** Log-log plot of glacier altitudinal range versus area for different regions. Solid lines are re-plots of the nonlinear regression curves fitted to the raw data.

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